Embedding Based Models – Lecture 17.2

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  E.g., predicting the rating for a person on a movie (collaborative filtering).
- Knowledge graphs: predict subject-verb-object triples.

Learning a relation between two entities

A relation between users and items (movies). From Movielens:

User	Item	Rating	Timestamp
196	242	3	881250949
186	302	3	891717742
22	377	1	878887116
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$\widehat{r_{ui}}$ = predicted rating of user u on item i

Es = set of (u, i, r) tuples in the training set (ignoring timestamp)

Minimize sum squares error:

$$\sum_{(u,i,r)\in Es} (\widehat{r_{ui}} - r)^2$$

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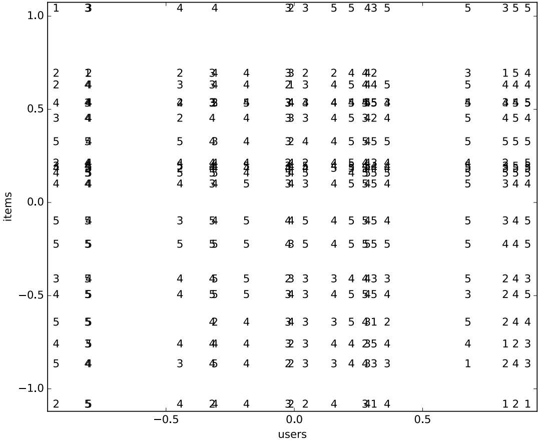
  - What about negative values?

What is being learned? (Single latent feature)

for each (u, i, r): r is plotted at point $(f_1[u], f_2[i])$.



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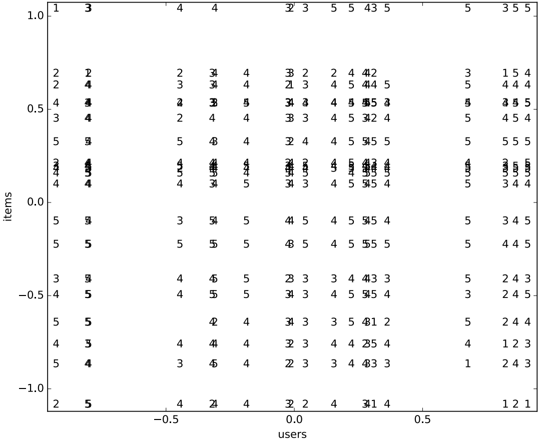

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What pattern would you expect?

Learning Relational Models with Multiple Latent Properties

- k latent features (Python notation):

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- Regularize parameters except μ.

Regularizing

- $L2$ regularization, minimize:

$$\left(\sum_{(u,i,r)\in Es} (\widehat{r_{ui}} - r)^2\right) + \lambda \sum_{parameter\ \ p} p^2$$

$L2$ regularization

Minimize:

$$\left(\sum_{(u,i,r)\in Es} (\mu + b_1[u] + b_2[i] + \sum_k E_1[u][f] * E_2[i][f] - r)^2 \right) + \lambda \left(\sum_i (b_1[u]^2 + \sum_f E_1[u][f]^2) + \sum_u (b_2[i]^2 + \sum_f E_2[i][f]^2) \right)$$

where λ is a regularization parameter.

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where λ is a regularization parameter.

To find minimizing parameters:

- Gradient descent
- Iterative least squares: fix one of E_1 and E_2; the problem is ridge regression in the other.

$\mu :=$ average rating
assign $E_1[u][f]$, $E_2[i][f]$ randomly and assign $b_1[i], b_2[u]$ arbitrarily
repeat:
    for each $(u, i, r) \in Es$:
        $e := \mu + b_1[i] + b_2[u] + \sum_k E_1[u][f] * E_2[i][f] - r$
        $b_1[i] := b_1[i] - \eta * e$
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        for each feature f:
            $E_1[u][f] := E_1[u][f] - \eta * e * E_2[i][f]$
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- **for each** item i:
  - $b_1[i] := b_1[i] - \eta * \lambda * b_1[i]$
  - **for each** feature f:
    - $E_1[u][f] := E_1[u][f] - \eta * \lambda * E_1[u][f]$
- **for each** user u:
  - $b_2[u] := b_2[u] - \eta * \lambda * b_2[u]$
  - **for each** feature k:
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    but the average probability is $1/k$, which is not derived from the data.

Knowledge Graphs and Triples

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All embeddings $E_1[e]$, $E_2[r]$ and $E_3[e]$ are the same length.

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- But, not all relations have same number of negative examples, eg. "*married-to*" vs "*has-streamed*".

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- Solution: also represent $(m, likes^{-1}, u)$ and $(d, directed\_by^{-1}, m)$.
- polyadic decomposition with inverses:

$$\widehat{p}(h, r, t) = \frac{1}{2}(\widehat{pd}(h, r, t) + \widehat{pd}(t, r^{-1}, h))$$

where $\widehat{pd}$ is the prediction from the polyadic decomposition.

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- The addition lets these values be combined.

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- The polyadic decomposition makes predictions by clustering entities in different ways.
- It learns about each entity; embeddings are used to predict interactions.
- It does not learn general knowledge that can be applied to other populations.