From Relations to Random Variables and Features – Lecture 17.1

Topics:

- Reconciling relations and random variables / features
- From knowledge graphs to random variables
- More general relationships

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Real World

What is the real world made of?

- A Features or random variables
- B Words, pixels, phonemes . . .
- C Entities and events (e.g., plants, people, diseases, lectures, university course)
- D Huh? There is a real world?

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  - ▶ The probability of Shakira
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- The word "variable" has different meanings in probability and logic.
  - ▶ In logic a variable denotes an entity.
  - ▶ In probability a variable denotes a function over possible worlds (that we may be uncertain of the value of).

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  E.g, predict what rating a user will give a movie
- Predicting identity, whether descriptions denote the same entity
  E.g., which citations refer to the same paper
- Predicting existence, whether an entity exists that fits a description
  E.g., whether there is a person in a particular room

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  E.g., relation *has-streamed* relation between person and a musical artist

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The functional case is treated as a relation of $k - 1$ arguments, with a non-Boolean prediction.