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  - ▶ Bob:  $Y \succ Z \succ X$
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**Condorcet paradox**
  - Surely this is just an outlier!!

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**Arrow's Theorem:** If there are three or more outcomes, these properties cannot simultaneously hold for any social preference function.

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A **mechanism** specifies the actions available to each agent and the outcomes of each action profile.

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- A mechanism should give the best outcome aggregated over all of the agents. A mechanism is **economically efficient** if the outcome chosen is one that maximizes the sum of the utilities of the agents.

## Example: meeting scheduler

Two mechanisms for scheduling meetings:

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Are these dominant-strategy truthful?

Alice, Bob, and Cory have to decide whether to meet on Monday, Tuesday, or Wednesday, with the following utilities for the meeting days:

	Monday	Tuesday	Wednesday
Alice	0	8	10
Bob	3	4	0
Cory	11	7	6

Should Alice be honest?

**Gibbard–Satterthwaite theorem** as long as there are three or more outcomes, the only mechanisms with dominant strategies have a **dictator**: an agent whose preferences determine the outcome.

# Vickrey–Clarke–Groves mechanism

- Introduce **money**, so that, for any two outcomes  $o_1$  and  $o_2$ , for each agent there is some (possibly negative) amount  $d$  such that the agent is indifferent between the outcomes  $o_1$  and  $o_2 + d$ .



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- VCG mechanism: Agents pay according to how much their participation affects the outcome.
- Agent  $i$  pays the sum of the value for the *other* agents if  $i$  had not participated minus the sum of the values for the other agents if  $i$  had participated.

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The VCG mechanism is both economically efficient and dominant-strategy truthful, assuming that agents only care about their utility and not about other agents' utilities or other agents' payments.

## Example: meeting scheduler

VCG Mechanism:

	Monday	Tuesday	Wednesday	Payment	Net Value
Alice	0	8	10		
Bob	3	4	0		
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Total	14	19	16		

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- What happens with payments?



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# Auctions

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- According to the VCG mechanism, the top bidder should get the item and pay the value of the second-highest bid. This is known as a **second-price auction**.
- This is equivalent (up to bidding increments) to having an **ascending auction**, where people specify how much they want to pay as a **proxy bid**; an agent converts proxy bids into real bids.