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- Agents that reason and act autonomoulsly can't be modeled as nature.

Multi-agent framework

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- Each agent's value depends on the outcome.

Normal Form of a Game

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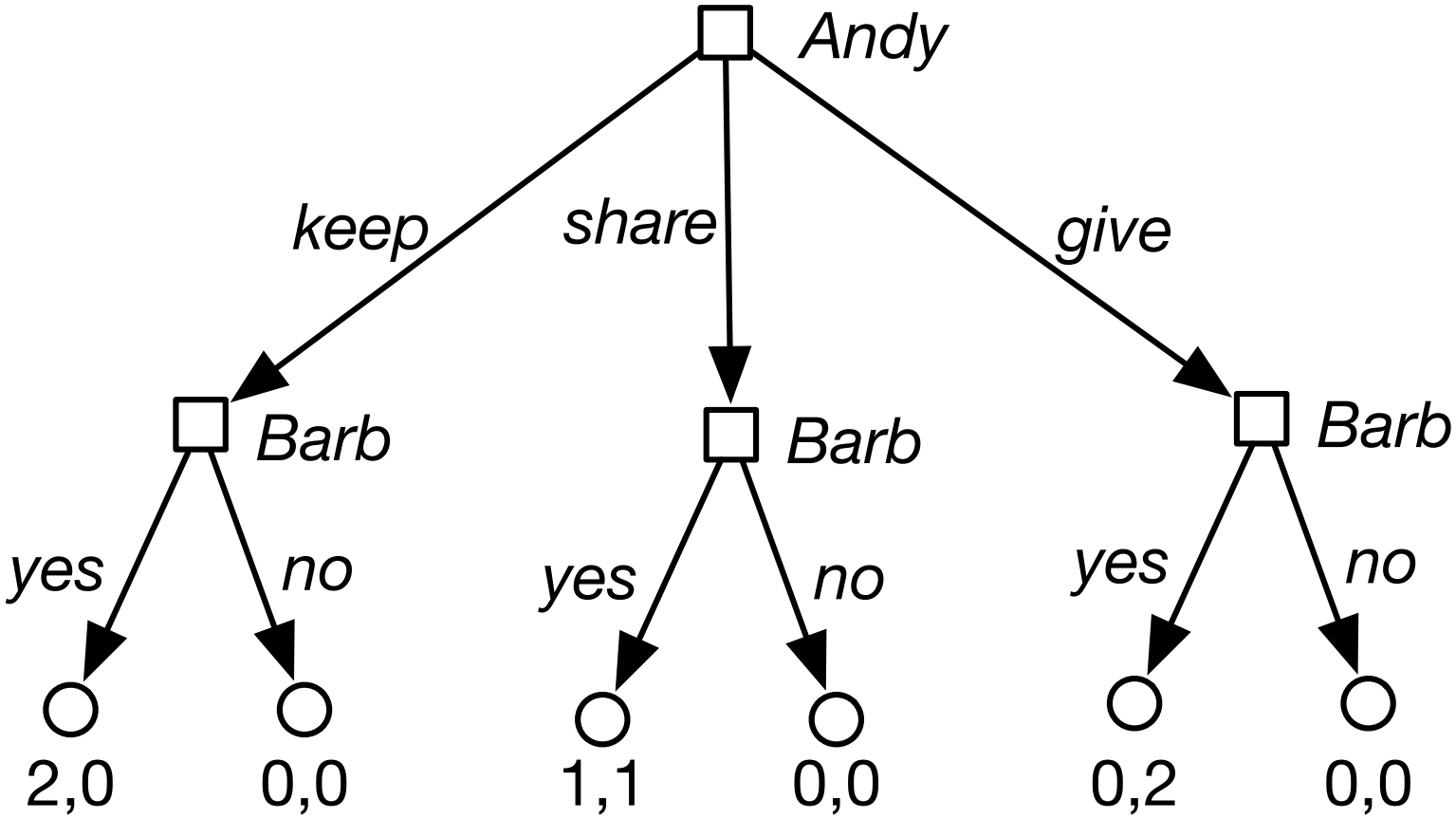
  An action profile σ is a tuple $\langle a_1, \ldots, a_n \rangle$, means agent i carries out a_i.
- a utility function $utility(\sigma, i)$ for action profile σ and agent $i \in I$, gives the expected utility for agent i when all agents follow action profile σ.

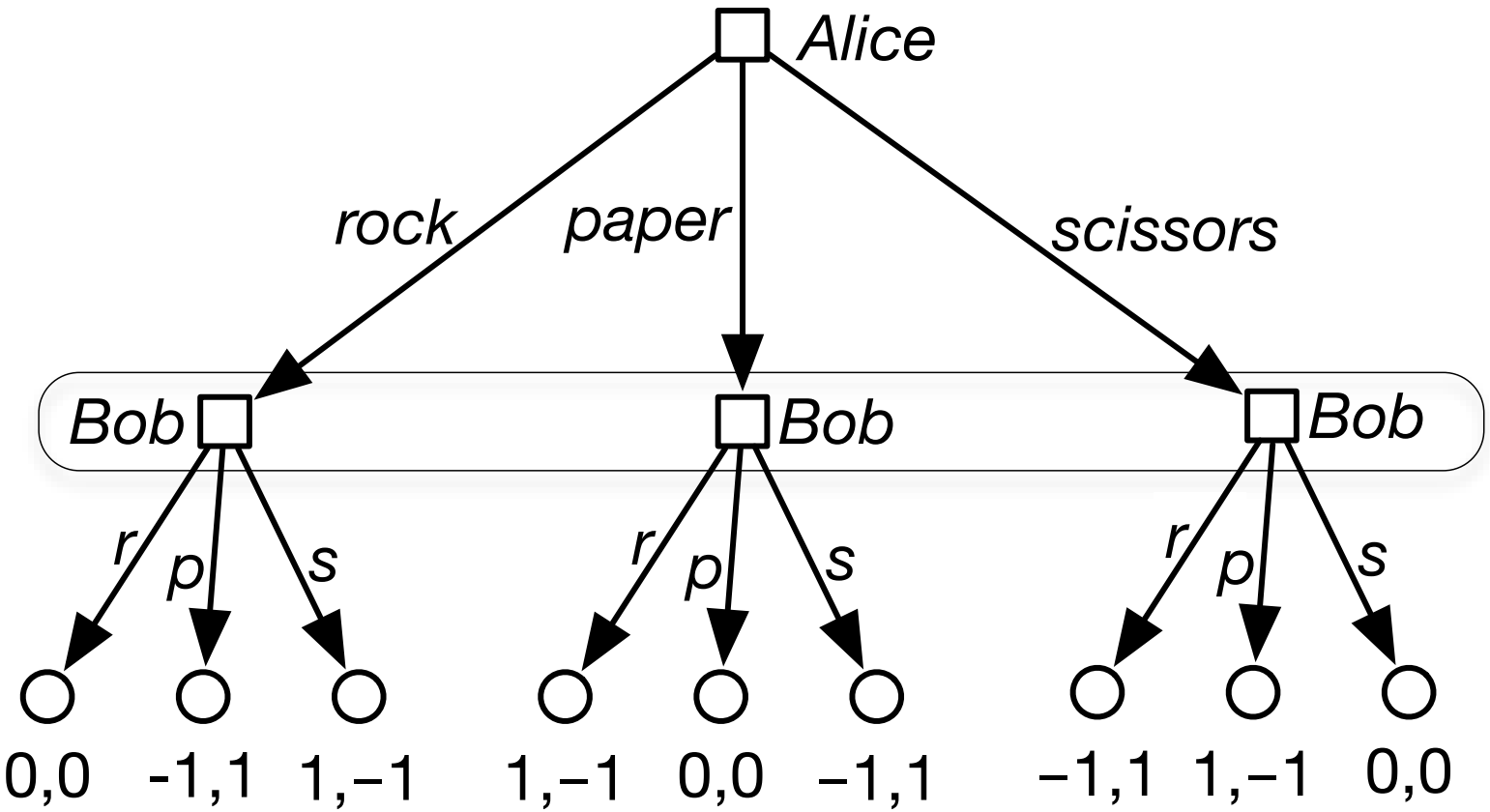
Rock-Paper-Scissors

		Bob		
		rock	*paper*	*scissors*
Alice	*rock*	$0, 0$	$-1, 1$	$1, -1$
	paper	$1, -1$	$0, 0$	$-1, 1$
	scissors	$-1, 1$	$1, -1$	0,0

Extensive Form of a Game

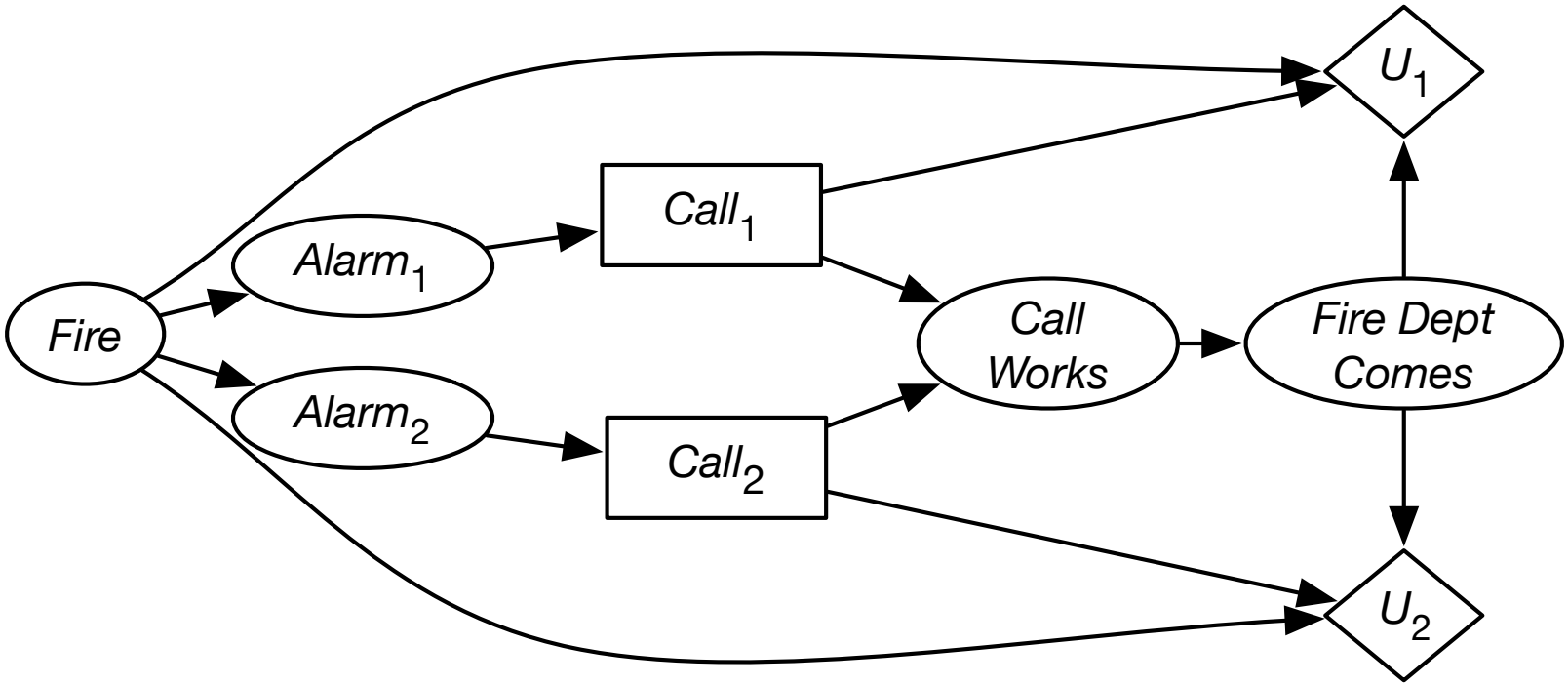


Extensive Form of an imperfect-information Game



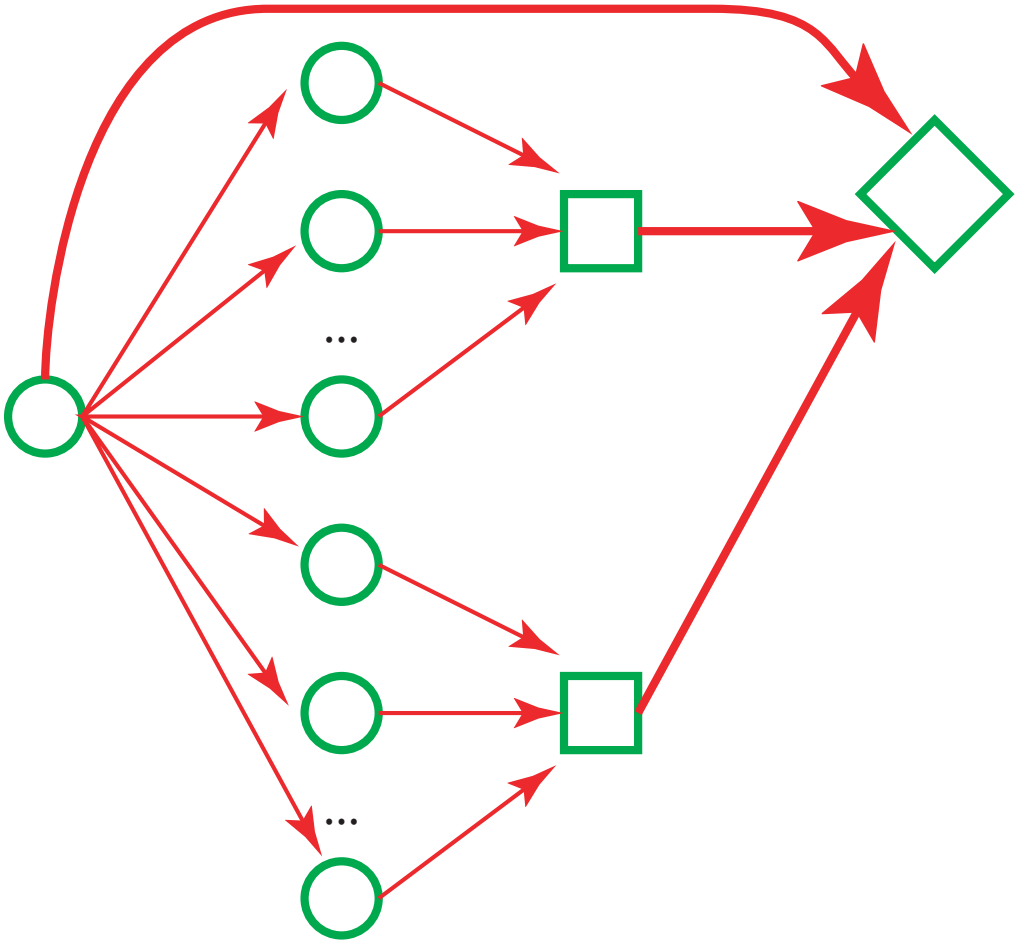
Bob cannot distinguish the nodes in an information set.

Multiagent Decision Networks



Value node for each agent.
Each decision node is owned by an agent.
The parents of each decision node specify what that agent will observe when making the decision

Multiple Agents, shared value



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- A strategy profile σ has a utility for each agent.
  Let $utility(\sigma, i)$ be the utility of strategy profile σ for agent i.
- If σ is a strategy profile:
  σ_i is the strategy of agent i in σ,
  σ_{-i} is the set of strategies of the other agents.
  Thus σ is $\sigma_i \sigma_{-i}$

Nash Equilibria

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- Theorem [Nash, 1950] Every finite game has at least one Nash equilibrium.

Multiple Equilibria

Hawk-Dove Game:

		Agent 2	
		dove	hawk
Agent 1	dove	R/2,R/2	0,R
	hawk	R,0	-D,-D

D and R are both positive with $D >> R$.

Coordination

Just because you know the Nash equilibria doesn't mean you know what to do:

		Agent 2	
		shopping	football
Agent 1	shopping	2,1	0,0
	football	0,0	1,2

Prisoner's Dilemma

Two strangers are in a game show. They each have the choice:

- Take \$100 for yourself
- Give \$1000 to the other player

This can be depicted as the playoff matrix:

		Player 2	
		take	give
Player 1	take	100,100	1100,0
	give	0,1100	1000,1000

Tragedy of the Commons

Example:

- There are 100 agents.
- There is an common environment that is shared amongst all agents. Each agent has 1/100 of the shared environment.
- Each agent can choose to do an action that has a payoff of +10 but has a -100 payoff on the environment
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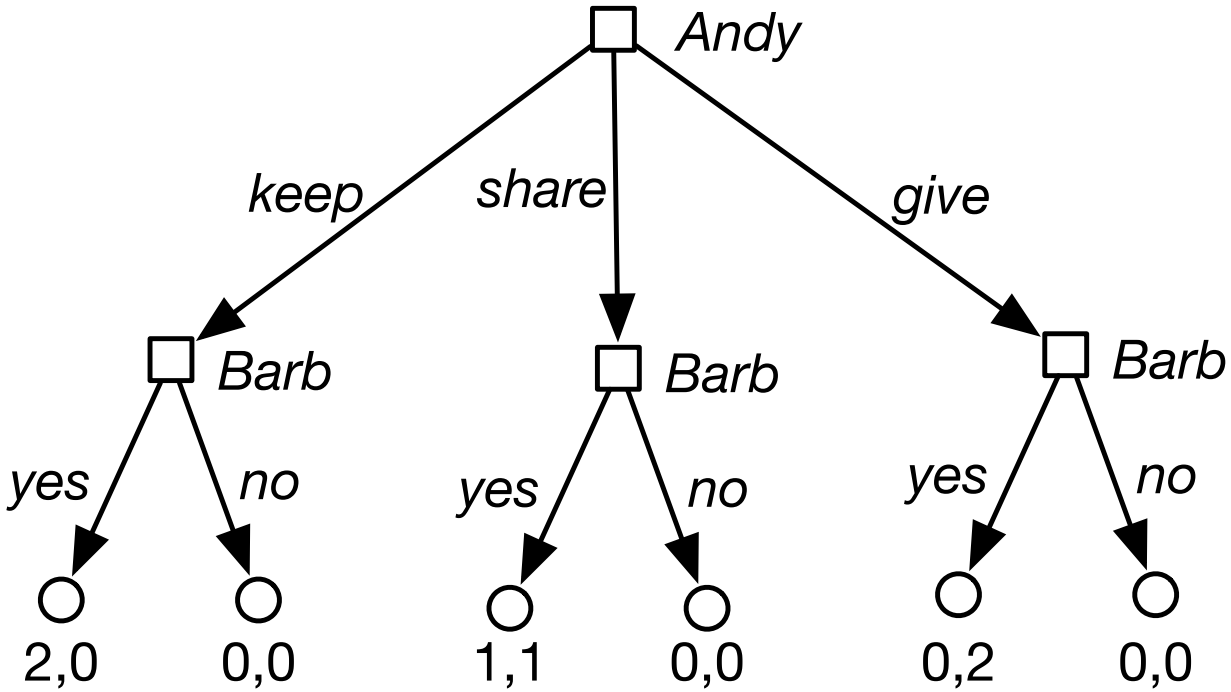

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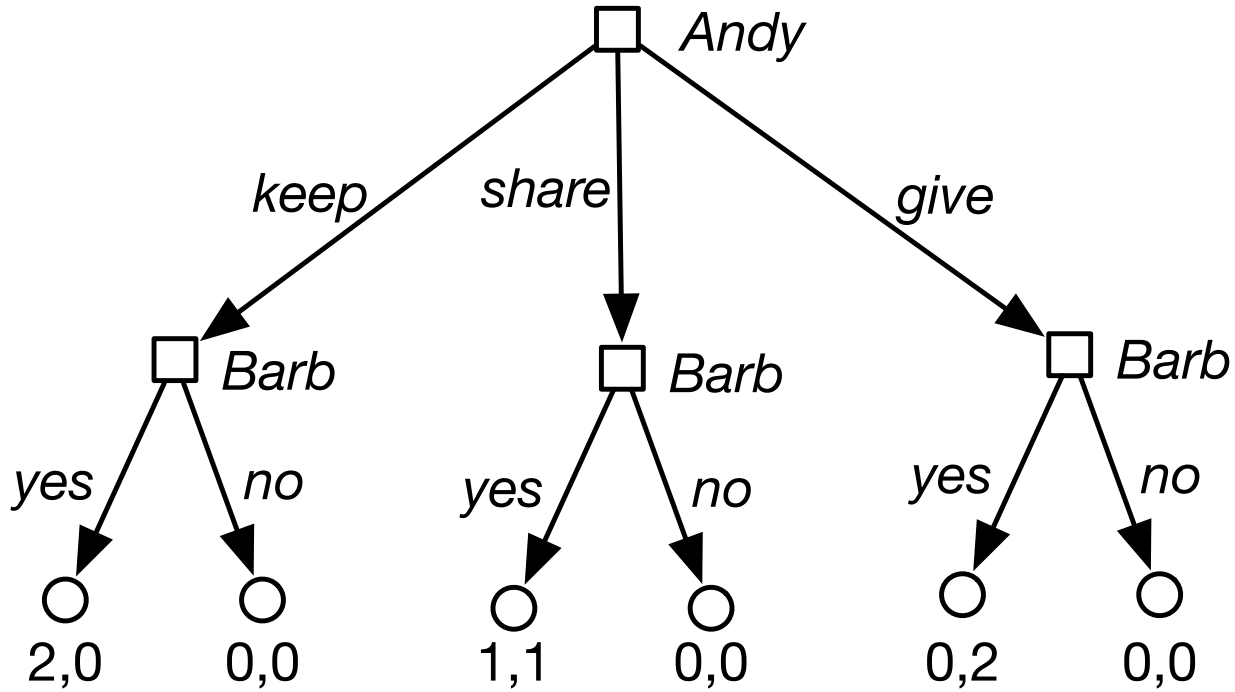

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What if the 2,0 payoff was 1.9,0.1?