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- Why?

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- This could be bad if the exploration policy is dangerous.
- On-policy learning learns the value of the policy being followed.
  e.g., act greedily 80% of the time and act randomly 20% of the time
- Why? If the agent is actually going to explore, it may be better to optimize the actual policy it is going to do.
- SARSA uses the experience $\langle s, a, r, s', a' \rangle$ to update $Q[s, a]$.

SARSA

initialize $Q[S, A]$ arbitrarily
observe current state s
select action a
repeat forever:
    carry out action a
    observe reward r and state s'
    select action a' using a policy based on Q
    $Q[s, a] :=$

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  $Q[s, a] := Q[s, a] + \alpha\,(r + \gamma Q[s', a'] - Q[s, a])$
  $s := s'$
  $a := a'$

Q-learning with Action Replay

initialize $Q[S, A]$ arbitrarily
$E = \{\}$
observe current state s
select action a
repeat forever:
  carry out action a
  observe reward r and state s'
  $E := E \cup \{\langle s, a, r, s' \rangle\}$
  $Q[s, a] :=$

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  $Q[s, a] := Q[s, a] + \alpha\,(r + \gamma \max_{a'} Q[s', a'] - Q[s, a])$
  repeat for a while:
    select $\langle s_1, a_1, r_1, s_1' \rangle \in E$
    $Q[s_1, a_1] :=$

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Model-based Reinforcement Learning

- Model-based reinforcement learning uses the experiences in a more effective manner.
- It is used when collecting experiences is expensive (e.g., in a robot or an online game); an agent can do lots of computation between each experience.

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- It is used when collecting experiences is expensive (e.g., in a robot or an online game); an agent can do lots of computation between each experience.
- Idea: learn the MDP and interleave acting and planning.
- After each experience, update probabilities and the reward, then do some steps of asynchronous value iteration.

Model-based learner

Data Structures: $Q[S, A]$, $T[S, A, S]$, $C[S, A]$, $R[S, A]$
Assign Q, R arbitrarily, $C = 0$, $T = 0$
observe current state s
repeat forever:
    select and carry out action a
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$T[s, a, s'] := T[s, a, s'] + 1$
$C[s, a] := C[s, a] + 1$
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$$Q[s_1, a_1] := R[s_1, a_1] + \sum_{s_2} \frac{T[s_1, a_1, s_2]}{C[s_1, a_1]} \left(\gamma \max_{a_2} Q[s_2, a_2]\right)$$

    $s := s'$

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What goes wrong with this?

Reinforcement Learning with Features

- Usually we don't want to reason in terms of states, but in terms of features.
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Reinforcement Learning with Features

- Usually we don't want to reason in terms of states, but in terms of features.
- In state-based methods, information about one state cannot be used by similar states.
- If there are too many parameters to learn, it takes too long.
- Idea: Express the value (Q) function as a function of the features. Most typical is a linear function of the features, or a neural network.

Reinforcement Learning

- flat or modular or hierarchical
- explicit states or features or individuals and relations
- static or finite stage or indefinite stage or infinite stage
- fully observable or partially observable
- deterministic or stochastic dynamics
- goals or complex preferences
- single agent or multiple agents
- knowledge is given or knowledge is learned
- perfect rationality or bounded rationality

SARSA with Generalization

controller *SARSA_with_Generalization*(*Learner*, γ)
Inputs
Learner with operations *Learner.add*(x, y) and *Learner.predict*(x).
$\gamma \in [0, 1]$: discount factor

SARSA with Generalization

controller $SARSA\_with\_Generalization(Learner, \gamma)$
    Inputs
        $Learner$ with operations $Learner.add(x, y)$ and $Learner.predict(x)$.
        $\gamma \in [0, 1]$: discount factor
    observe current state s
    select action a
    repeat
        $do(a)$
        observe reward r and state s'

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  observe current state s

  select action a

  repeat

    $do(a)$

    observe reward r and state s'

    select action a' based on $Learner.predict((s', a'))$

    $Learner.add((s, a), r + \gamma * Learner.predict((s', a')))$

SARSA with Generalization

```
controller SARSA_with_Generalization(Learner, γ)
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        γ ∈ [0, 1]: discount factor
    observe current state s
    select action a
    repeat
        do(a)
        observe reward r and state s′
        select action a′ based on Learner.predict((s′, a′))
        Learner.add((s, a), r + γ ∗ Learner.predict((s′, a′)))
        s := s′
        a := a′
    until termination
```

Review: Gradient descent

To find a (local) minimum of a real-valued function $f(x)$:

- assign an arbitrary value to x
- repeat

$$x :=$$

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where η is the step size

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Review: Linear Regression

- A linear function of variables $x_1, \ldots, x_n$ is of the form

$$f^{\overline{w}}(x_1, \ldots, x_n) = w_0 + w_1 x_1 + \cdots + w_n x_n$$

$\overline{w} = \langle w_0, w_1, \ldots, w_n \rangle$ are weights. (Let $x_0 = 1$).

- Given a set E of examples.
  Example e has input $x_i = e_i$ for each i and observed value, o_e:

$$Error_E(\overline{w}) = \sum_{e \in E} (o_e - f^{\overline{w}}(e_1, \ldots, e_n))^2$$

- Minimizing the error using gradient descent, each example should update w_i using:

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$$w_i := w_i - \eta \frac{\partial Error_E(\overline{w})}{\partial w_i}$$

Review: Gradient Descent for Linear Regression

Given E: set of examples over n features
each example e has inputs $(e_1, \ldots, e_n)$ and output o_e:
Assign weights $\overline{w} = \langle w_0, \ldots, w_n \rangle$ arbitrarily
repeat:
For each example e in E:
let $\delta = o_e - f^{\overline{w}}(e_1, \ldots, e_n)$
For each weight w_i:
$w_i := w_i + \eta \delta e_i$

SARSA with linear function approximation

- One step backup provides the examples that can be used in a linear regression.
- Suppose $F_1, \ldots, F_n$ are the features of the state and the action.
- So $Q_{\overline{w}}(s,a) = w_0 + w_1 F_1(s,a) + \cdots + w_n F_n(s,a)$
- An experience $\langle s, a, r, s', a' \rangle$ provides the "example":
  - old predicted value:
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- An experience $\langle s, a, r, s', a' \rangle$ provides the "example":
  - old predicted value: $Q_{\overline{w}}(s, a)$
  - new "observed" value: $r + \gamma Q_{\overline{w}}(s', a')$
- Treat $r + \gamma Q_{\overline{w}}(s', a')$ as a new training example for $Q(s, a)$ in linear regression (or other supervised learning algorithm).

SARSA with linear function approximation

Given γ:discount factor; η:step size
Assign weights $\overline{w} = \langle w_0, \ldots, w_n \rangle$ arbitrarily
observe current state s
select action a
repeat forever:
    carry out action a
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  let $\delta = r + \gamma Q_{\overline{w}}(s', a') - Q_{\overline{w}}(s, a)$
  For $i = 0$ to n
    $w_i := w_i + \eta \delta F_i(s, a)$

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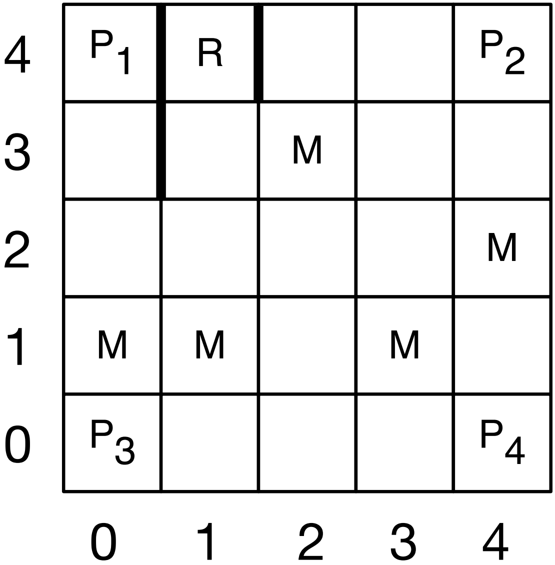
    let $\delta = r + \gamma Q_{\overline{w}}(s', a') - Q_{\overline{w}}(s, a)$
    For $i = 0$ to n
        $w_i := w_i + \eta \delta F_i(s, a)$
    $s := s'$
    $a := a'$

Monster Game



Example Features

- $F_1(s, a) = 1$ if a goes from state s into a monster location and is 0 otherwise.
- $F_2(s, a) = 1$ if a goes into a wall, is 0 otherwise.
- $F_3(s, a) = 1$ if a goes toward a prize.
- $F_4(s, a) = 1$ if the agent is damaged in state s and action a takes it toward the repair station.
- $F_5(s, a) = 1$ if the agent is damaged and action a goes into a monster location.
- $F_6(s, a) = 1$ if the agent is damaged.
- $F_7(s, a) = 1$ if the agent is not damaged.
- $F_8(s, a) = 1$ if the agent is damaged and there is a prize in direction a.
- $F_9(s, a) = 1$ if the agent is not damaged and there is a prize in direction a.

Example Features

- $F_{10}(s, a)$ is the distance from the left wall if there is a prize at location P_0, and is 0 otherwise.
- $F_{11}(s, a)$ has the value $4 - x$, where x is the horizontal position of state s if there is a prize at location P_0; otherwise is 0.
- $F_{12}(s, a)$ to $F_{29}(s, a)$ are like F_{10} and F_{11} for different combinations of the prize location and the distance from each of the four walls.
  For the case where the prize is at location P_0, the y-distance could take into account the wall.

Problems and Variants of function approximation

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- Use the policy to do more than one-step lookahead (better estimate of $Q(s', a')$).
  For example, compute expected value by generating samples of the rest of a game.

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- Idea:
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  - at each generation, the best controllers are combined to form a new population of controllers
- Performance is sensitive to representation of controller, and ways to combine them.