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The theoretical properties of the exploration-exploitation tradeoff are often studied in for bandits.
(A one-armed bandit is slot-machine / poker-machine.)
Each machine has its own distribution of payouts.
The action is to choose which machine to play;
— the agent repeatedly chooses an action from the same state.

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  Problem:
  - Very bad actions get selected as much as promising actions that are not maximal.

Softmax Exploration

- Actions with a higher Q-value are more likely to be selected.
  Softmax action selection: in state s, choose a with probability

$$\frac{e^{Q[s,a]/\tau}}{\sum_a e^{Q[s,a]/\tau}}$$

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τ	$e^{1/\tau}$
10	1.105
1	2.718
0.1	22026.5

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- Softmax selection doesn't take into account how many times an action has been tried, which affects how good the Q estimate is.
- The upper confidence bound is an estimate of the expected value such that it is be very unlikely that the actual value is greater than this.
- The upper confidence bound UCB1 is:

$$UCB1(s, a) = Q[s, a] + C * \sqrt{\frac{\log N(s)}{N[s, a]}}$$

  where
  - $N[s, a]$ is how many times action a has been selected in state s
  - $N(s) = \sum_a N[s, a]$ is how many times state s has been visited.
  - C is a constant that depends on the magnitude of the Q-values. If the values are all in range [0,1], then $C = \sqrt{2}$ has good theoretical properties
- A agent chooses action a with the highest $UCB1(s, a)$ value.

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- If the return is a real number, you could assume the distribution is a Gaussian, parameterized by the mean and the variance. For each state, choose the action a that maximizes

$$Q[s,a] + C * \frac{randn()}{\sqrt{N[s,a]}}$$

where $randn()$ returns a random number using the standard normal distribution (mean is 0, variance is 1).
C is chosen to reflect the scale of the Q-values.

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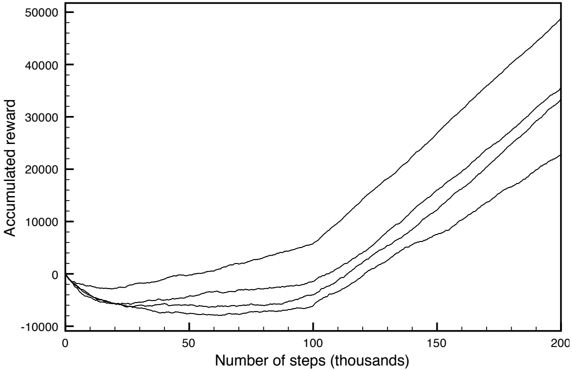
- How to update distribution given feedback? See Chapter 14.

Evaluating Reinforcement Learning Algorithms



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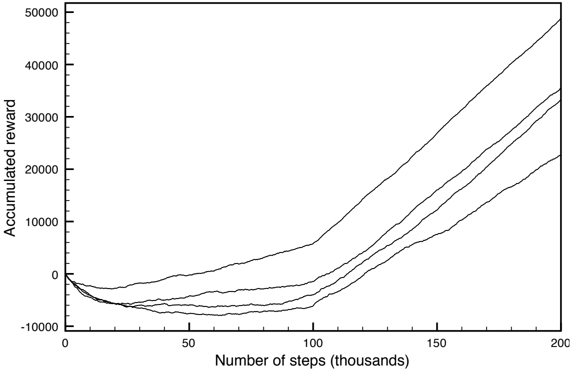

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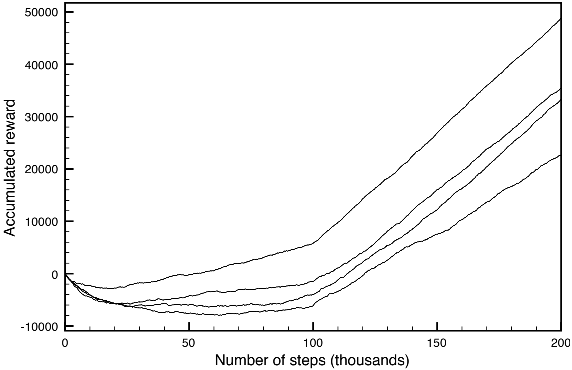

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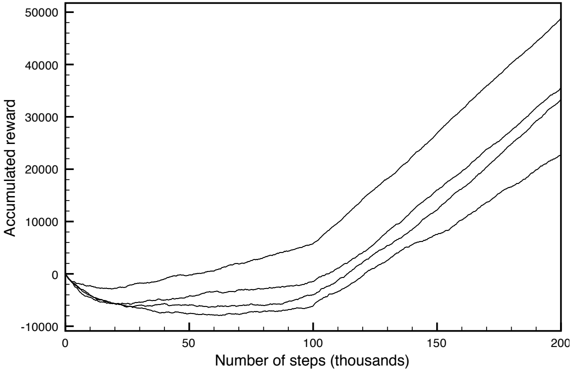

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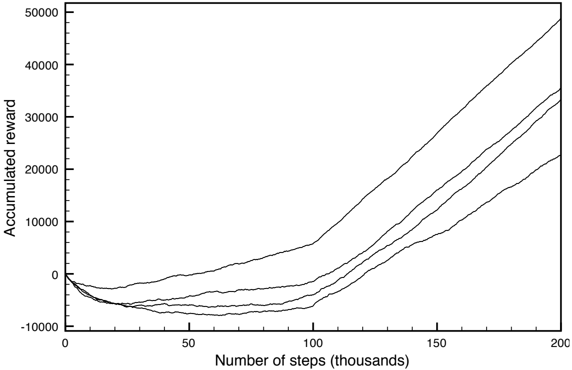

Each algorithm stops exploring at 100,000 steps.

- Alternative #1: plot mean reward received, but it is noisy
- Alternative #2: plot discounted reward for each time step, but it can only be evaluated in retrospect.