Making Decisions Under Uncertainty

What an agent should do depends on:

- The agent's ability — what options are available to it.
- The agent's beliefs — the ways the world could be, given the agent's knowledge.
  Sensing updates the agent's beliefs.
- The agent's preferences — what the agent wants and tradeoffs when there are risks.

Decision theory specifies how to trade off the desirability and probabilities of the possible outcomes for competing actions.

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- A possible world specifies a value for each decision variable and each random variable.
- For each assignment of values to *all* decision variables, there is a probability distribution over random variables.
- The probability of a proposition is undefined unless the agent conditions on the values of all decision variables.

Decision Tree for Delivery Robot

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The robot can choose to go the short way past the stairs or a long way that reduces the chance of an accident.

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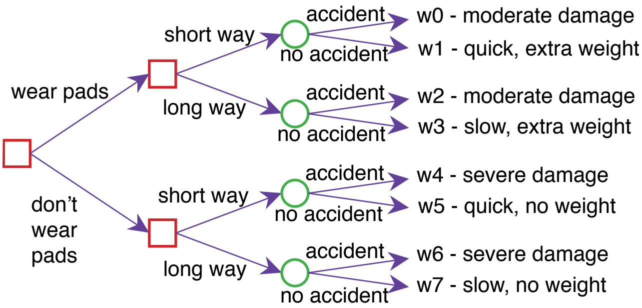

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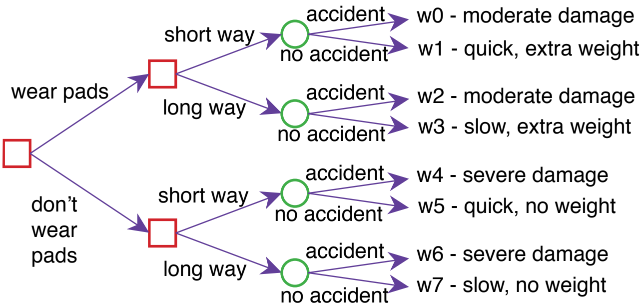

Square boxes represent decisions that the robot can make. Circles represent random variables that the robot can't observe before making its decision.

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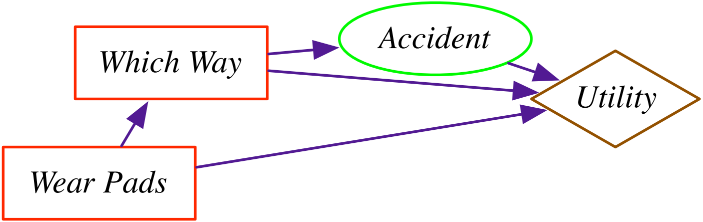

This shows explicitly which nodes affect whether there is an accident.

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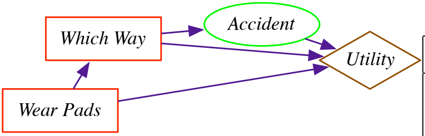
- (No tables associated with the decision nodes.)

Example Initial Factors



Which Way	Accident	Value
long	true	0.01
long	false	0.99
short	true	0.2
short	false	0.8

Which Way	Accident	Wear Pads	Value
long	true	true	30
long	true	false	0
long	false	true	75
long	false	false	80
short	true	true	35
short	true	false	3
short	false	true	95
short	false	false	100

Finding an optimal decision

- Suppose the random variables are $X_1, \ldots, X_n$, and utility depends on $V_{i_1}, \ldots, V_{i_k}$ (random and/or decision variables)

$$\mathbb{E}(u \mid D) \quad =$$

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  - This creates a factor on D that gives the expected utility for each value in the domain of D
  - Choose the D with the maximum value in the factor.

After summing out Accident

Which Way	Wear Pads	Value
long	true	74.55
long	false	79.2
short	true	83.0
short	false	80.6

(Sequential) Decision Networks

- flat or modular or hierarchical
- explicit states or features or individuals and relations
- static or finite stage or indefinite stage or infinite stage
- fully observable or partially observable
- deterministic or stochastic dynamics
- goals or complex preferences
- single agent or multiple agents
- knowledge is given or knowledge is learned
- perfect rationality or bounded rationality

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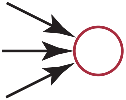
- Subsequent actions can depend on what is observed.
  What is observed depends on previous actions.
- Often the sole reason for carrying out an action is to provide information for future actions.
  For example: diagnostic tests, spying.

Sequential decision problems

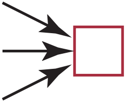
- A sequential decision problem consists of a sequence of decision variables $D_1, \ldots, D_n$.
- Each D_i has an information set of variables $parents(D_i)$, whose value will be known at the time decision D_i is made.

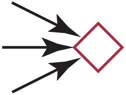
Decisions Networks

A decision network is a graphical representation of a finite sequential decision problem, with 3 types of nodes:



- A random variable is drawn as an ellipse. Arcs into the node represent probabilistic dependence. Each random variable has a domain and an associated factor.





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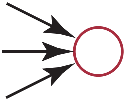

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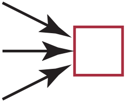

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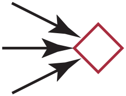

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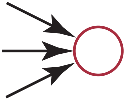

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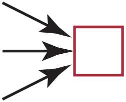

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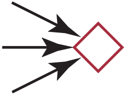
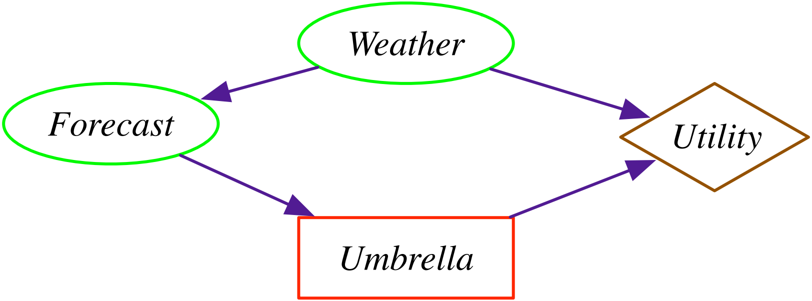


- A utility node is drawn as a diamond. Arcs into the node represent variables that the utility depends on. The utility node has no domain, and a factor on the parents of the node.

Umbrella Decision Network



- The agent has to decide whether to take its umbrella.
- It observes

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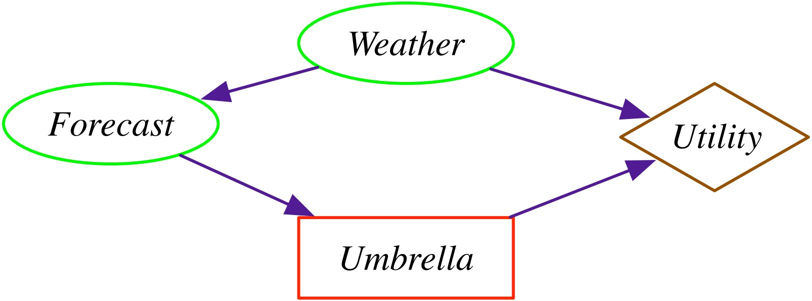

- The agent has to decide whether to take its umbrella.
- It observes the forecast.
- It doesn't observe

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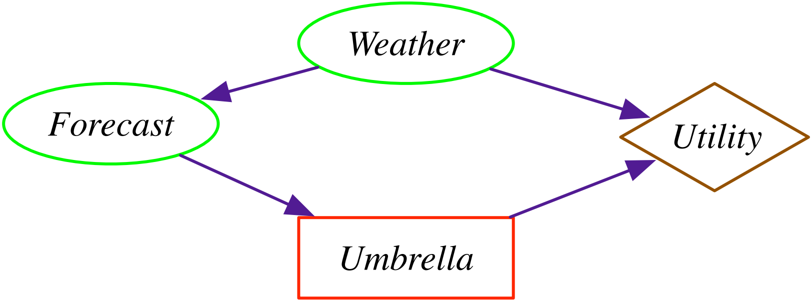

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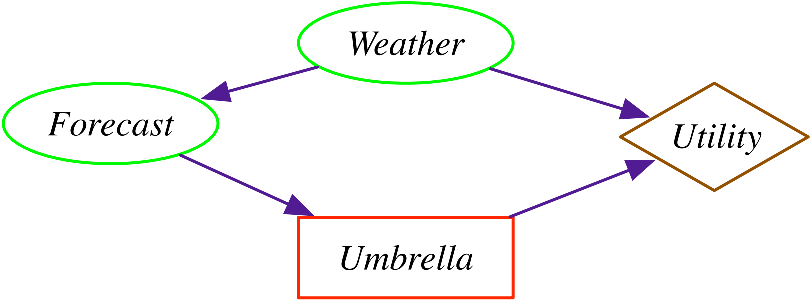

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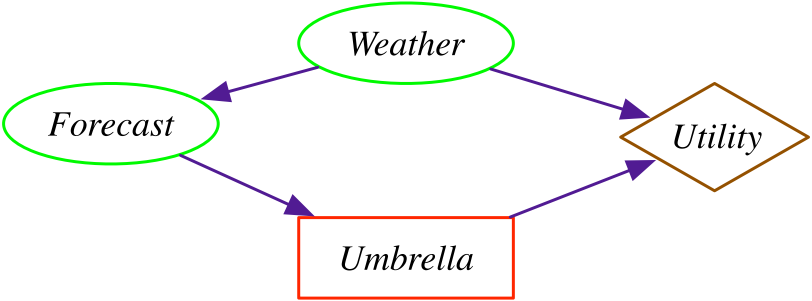

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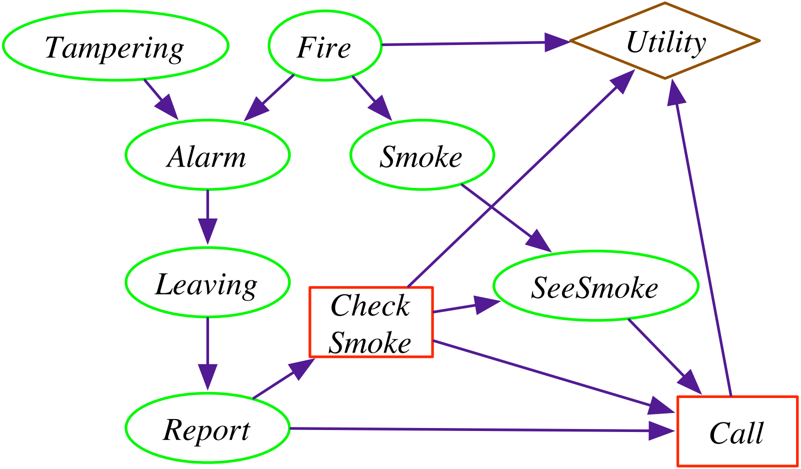
- The utility depends on the weather and whether the agent takes the umbrella.

Decision Network for the Alarm Problem



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- Any parent of a decision node is a parent of subsequent decision nodes. Thus the agent remembers its previous observations.

What should an agent do?

- What an agent should do at any time depends on what it will do in the future.
- What an agent does in the future depends on what it did before.

Policies

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- A policy is a sequence of decision functions; one for each decision node.

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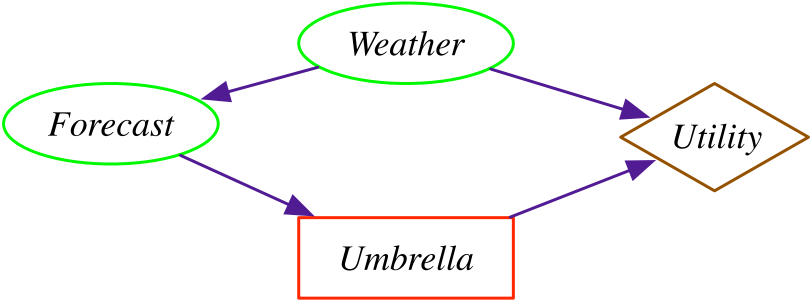

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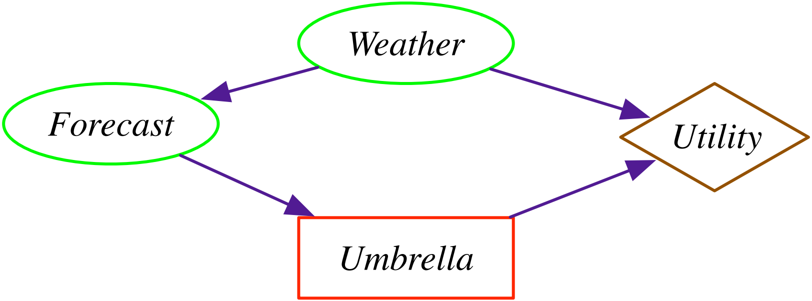

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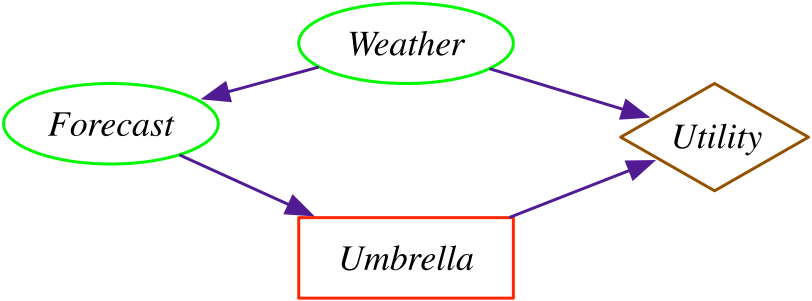

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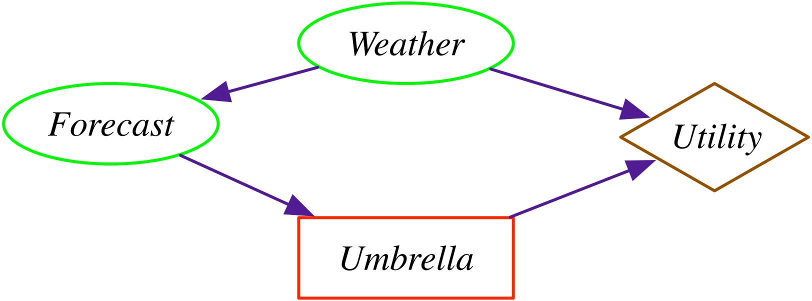

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There are $2^3 = 8$ policies

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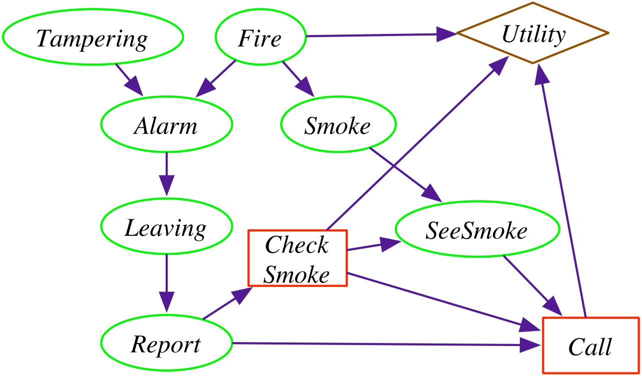

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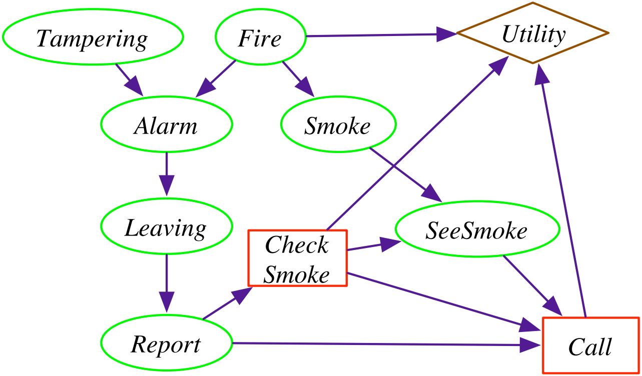
Decision Network for the Alarm Problem



All variables are Boolean. Some policies:

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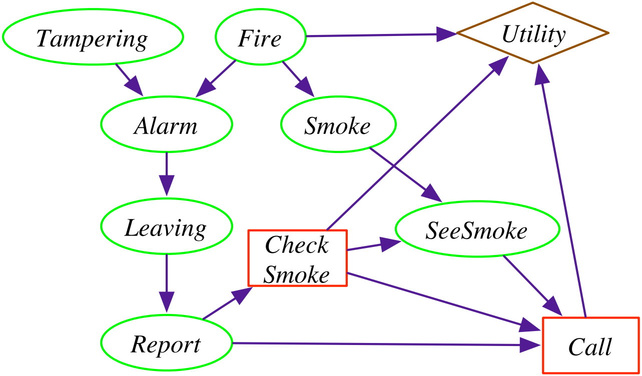

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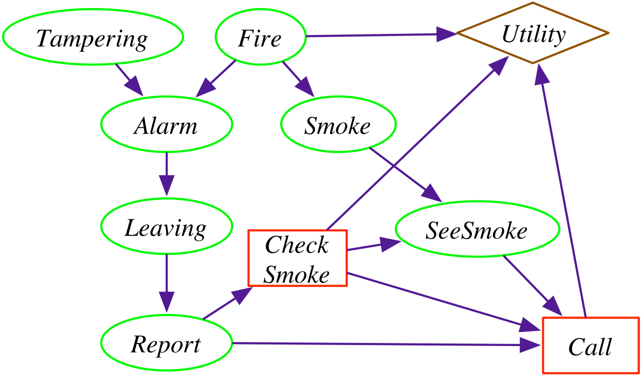

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There are $2^2 * 2^8 = 1024$ policies.

Finding an optimal policy

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  where $parents(X_i)_\pi$ include the values chosen by policy π.

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&= \sum_{X_1,\ldots,X_n} \prod_{i=1}^{n} P(X_i \mid parents(X_i)_\pi) \times u(V_{i_1}, \ldots, V_{i_k})
\end{aligned}
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where $parents(X_i)_\pi$ include the values chosen by policy π.

Idea:

  - Sum out all of the random variables to compute expected utility.

Finding an optimal policy

- Suppose the random variables are $X_1, \ldots, X_n$, and utility depends on $V_{i_1}, \ldots, V_{i_k}$ (random and/or decision variables)

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- To split on a decision node, return the maximum value of its children and the appropriate $(context, action)$

Depth-first search for optimizing decision network

procedure *DN_dfs*(*Con*, *Fs*, *Ds*) returns (*value*, *policy*)
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        return (eval(f, Con) ∗ v, p)
    else if Con assigns all parents in D ∈ Ds then
        max := −∞;   best := ⊥
        for val in domain(D) do
            (v, p) := DN_dfs(Con ∪ {D=val}, Fs, Ds \ {D})
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    $max := -\infty; \quad best := \perp$
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      if $v > max$ **then**
        $max := v; \quad best := \{\langle Con, D{=}val \rangle\} \cup p$
    return $(max, best)$
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        return (max, best)
    else
        select variable X that is parent of all D ∈ Ds
        for each value of X, recursivley call DN_dfs, and return
sum of values and union of policies
```

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`AIPython.org` decnNetworks.py

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- until there are no more decision nodes.
- Sum out the remaining random variables.
- Multiply the factors: this is the expected utility of an optimal policy.

Initial factors for the Umbrella Decision

Weather	Value
norain	0.7
rain	0.3

Weather	Fcast	Value
norain	sunny	0.7
norain	cloudy	0.2
norain	rainy	0.1
rain	sunny	0.15
rain	cloudy	0.25
rain	rainy	0.6

Weather	Umb	Value
norain	take	20
norain	leave	100
rain	take	70
rain	leave	0

. . . first sum out *Weather*.

Eliminating By Maximizing

f:

Fcast	Umb	Val
sunny	take	12.95
sunny	leave	49.0
cloudy	take	8.05
cloudy	leave	14.0
rainy	take	14.0
rainy	leave	7.0

$\max_{Umb} f$:

Fcast	Val
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cloudy	14.0
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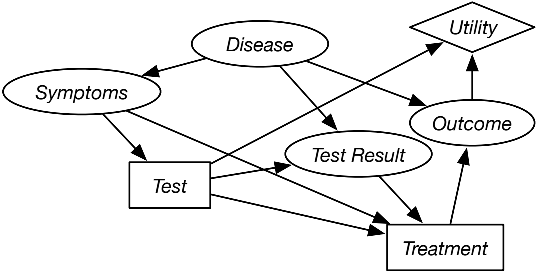
$\arg\max_{Umb} f$:

Fcast	Umb
sunny	leave
cloudy	leave
rainy	take

Exercise



- What are the factors?

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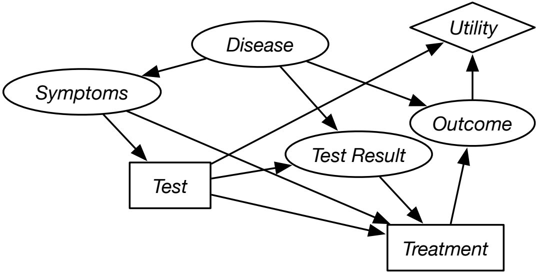

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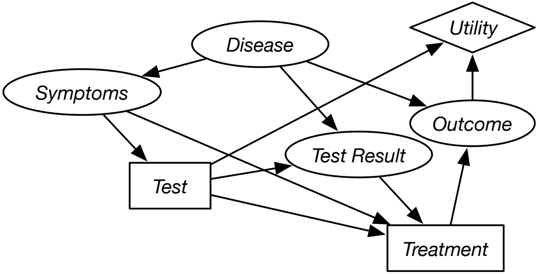

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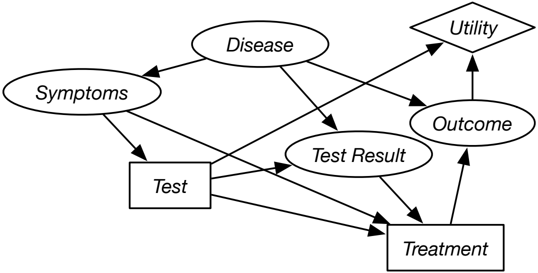

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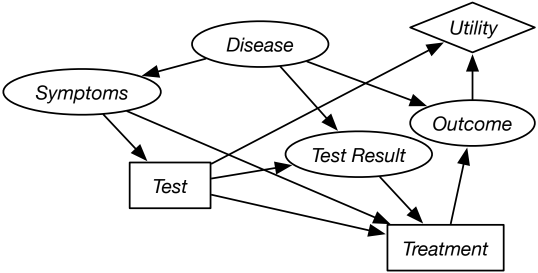

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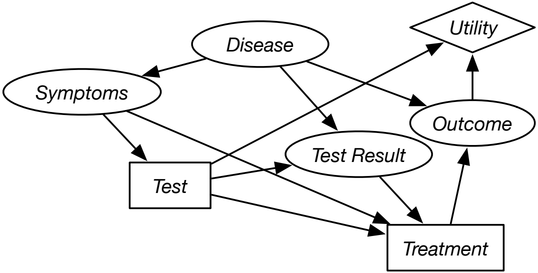

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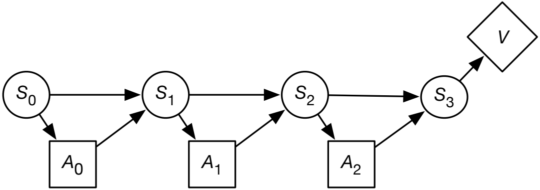
- What factors are created after maximization?

Chain



- Is this decision network no-forgetting?

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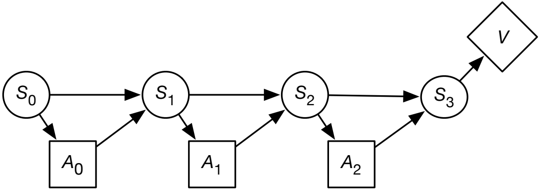

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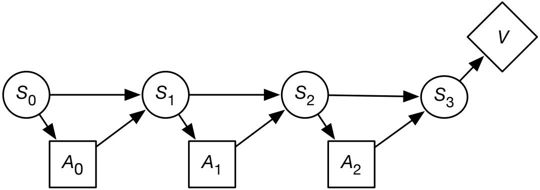

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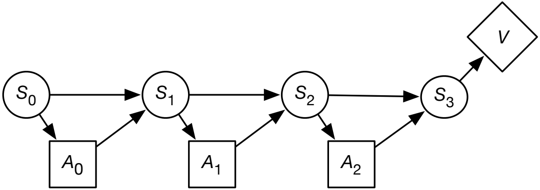

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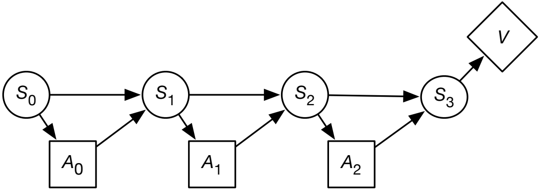

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`AIPython` decnNetworks.py ch3

Complexity of finding an optimal policy

Decision D has k binary parents, and has b possible actions:

- there are assignments of values to the parents.

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- there are   different decision functions.

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- there are $\prod_{i=1}^{n} b_i^{2^{k_i}}$ policies.
- optimizing in the variable elimination algorithm takes $$O\left(\sum_{i=1}^{n} b_i * 2^{k_i}\right)$$ time
- The dynamic programming algorithm is much more efficient than searching through policy space.

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- The value of information is always non-negative.
- It is positive only if the agent changes its action depending on X.
- The value of information provides a bound on how much an agent should be prepared to pay for a sensor. How much is a better weather forecast worth?
- We need to be careful when adding an arc would create a cycle. E.g., how much would it be worth knowing whether the fire truck will arrive quickly when deciding whether to call them?

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- You need to be explicit about what information is available when you control X.
- If you control X without observing, controlling X can be worse than observing X. E.g., controlling a thermometer.
- If you keep the parents the same, the value of control is always non-negative.