# Counterfactual Reasoning

- The do calculus is for intervening before observing.
  $P(x \mid y, do(z))$ means the probability of x after doing z then observing y.

- The other case is observing then intervening.

- When the intervention is different from what actually happened, this is counterfactual reasoning, which is asking "what if something else were true".

- Let's use a more general notion of counterfactual, where you can ask "what if x were true" without knowing whether x were true.

Example: firing squad
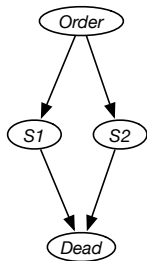
- A captain can give an order to a number of shooters who can each shoot to kill a prisoner condemned to death.
- Each shooter can think "I wasn't responsible for killing the prisoner, because the prisoner would be dead even if I didn't shoot".
- The captain some probability of issuing order.
- Each shooter obeys order with high probability.
- The prisoner is dead if any of the shooters shoot.
- One counterfactual is "if the second shooter shot, what would have happened if the second shooter had not shot?"
- Another counterfactual query is "if the prisoner died; what would have happened if shooter 2 had not shot".

Counterfactual Reasoning

$E=e$ is observed, the query is "what if $C=c$ happened?"

1. Determine what must be true for $E=e$ to be observed. This is an instance of abduction.
2. Intervene to make $C=c$ true.
3. Query the resulting model, using the posterior probabilities from the first step as the prior for the intervened model.

This can be implemented by constructing a causal network, from which queries from the counterfactual situation can be made.

Counterfactual Causal Network

To model observing $E=e$, and asking "what if $C=c$ happened":

- represent the problem using a causal network where conditional probabilities are in terms of a deterministic system with stochastic inputs, such as a probabilistic logic program or a probabilistic program
- create a node C' (a primed variable) with the same domain as C but with no parents
- for each descendant D of C in the original model, create a node D'
- The conditional probability for D' is the same as for D, but using primed parents that exist.
- Condition on $C'=c$
- Condition on the observations of the initial situation using unprimed variables.

Example

(a) (b) (c)
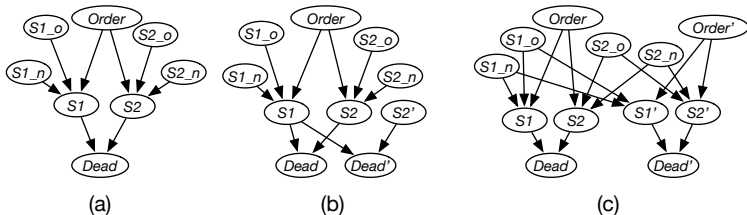
(a) original network, e.g., $s1 \leftrightarrow (order \wedge s1\_o) \vee (\neg order \wedge s1\_n)$

(b) "what if shooter 2 shot" or "what if shooter 2 didn't shoot".
"the prisoner is dead; what is the probability that the prisoner
would be dead if the second shooter did not shoot?":

$$P(dead' \mid dead \wedge \neg s2')$$

(c) "what if the order was not given" "shooter 1 didn't shoot and
the prisoner was dead; what is the probability the prisoner is
dead if the order was not given":

$$P(dead' \mid \neg s1 \wedge dead \wedge \neg order')$$