Stochastic Simulation

- Idea: probabilities $\leftrightarrow$ samples
- Get probabilities from samples:

X	$count$
x_1	n_1
$\vdots$	$\vdots$
x_k	n_k
$total$	m

$\leftrightarrow$

X	$probability$
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$\leftrightarrow$

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x_k	n_k/m

- If we could sample from a variable's (posterior) probability, we could estimate its (posterior) probability.

Generating samples from a distribution

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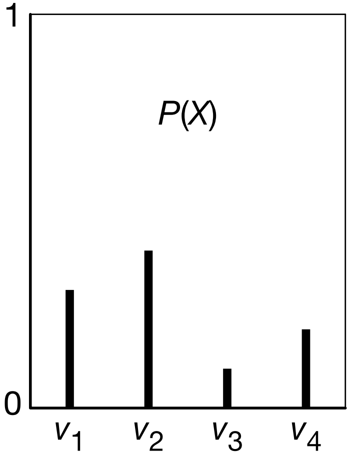
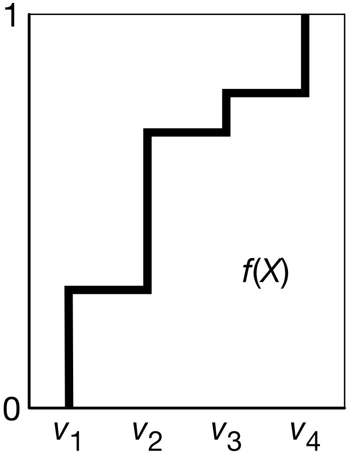
- Select a value y uniformly in the range $[0, 1]$.
- Select the x such that $f(x) = y$.

Cumulative Distribution





Hoeffding's inequality

Theorem (Hoeffding): Suppose p is the true probability, and s is the sample average from n independent samples; then

$$P(|s - p| > \epsilon) \leq 2e^{-2n\epsilon^2}.$$

Guarantees a probably approximately correct estimate of probability.

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ϵ	δ	n
0.1	0.05	185
0.01	0.05	18,445
0.1	0.01	265

Forward sampling in a belief network

- Sample the variables one at a time; sample parents of X before sampling X.
- Given values for the parents of X, sample from the probability of X given its parents.

Rejection Sampling

- To estimate a posterior probability given evidence $Y_1 = v_1 \wedge \ldots \wedge Y_j = v_j$:
- Reject any sample that assigns Y_i to a value other than v_i.

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- Reject any sample that assigns Y_i to a value other than v_i.
- The non-rejected samples are distributed according to the posterior probability:

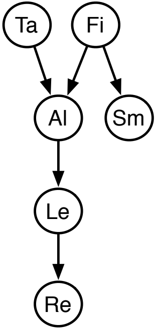
$$P(\alpha \mid evidence) \approx \frac{\sum_{\alpha \text{ is true in sample}} 1}{\sum_{sample} 1}$$

where we consider only samples consistent with evidence.

Rejection Sampling Example: $P(ta \mid sm, re)$

Observe $Sm = true, Re = true$

	Ta	Fi	Al	Sm	Le	Re
s_1	false	true	false	true	false	false



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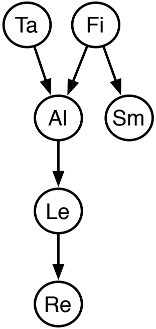

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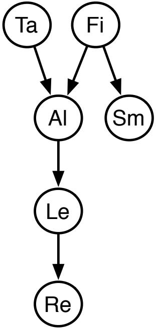

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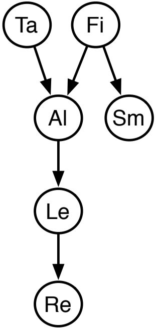

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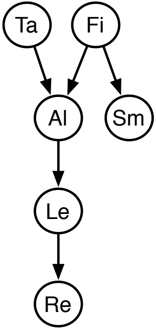

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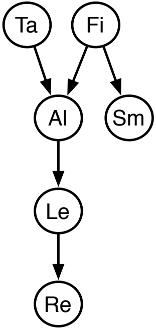

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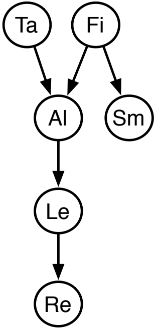

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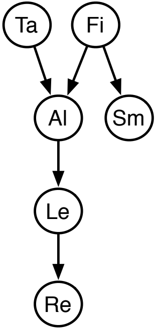

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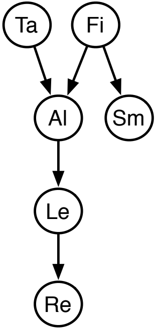

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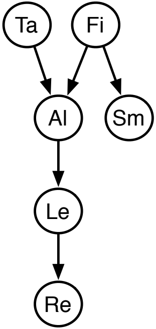

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$P(sm) = 0.02$
$P(re \mid sm) = 0.32$
There are 1000 samples.
How many are rejected?
How many are used?

Rejection Sampling Example: $P(ta \mid sm, re)$

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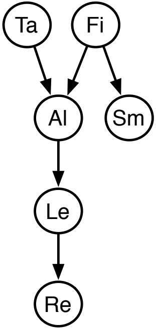

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How many are used?

Doesn't work well when evidence is unlikely.

Importance Sampling

- Samples have weights: a real number associated with each sample that takes the evidence into account.

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- Mix exact inference with sampling: don't sample all of the variables, but weight each sample according to $P(\mathit{evidence} \mid \mathit{sample})$.

Importance Sampling (Likelihood Weighting)

procedure *likelihood_weighting*(Bn, e, H, n):
  # Approximate $P(H \mid e)$ in belief network Bn using n samples.
  # H has some real domain (e.g., $\{0, 1\}$)
  $mass := 0$ # mass of all samples
  $hmass := 0$ # weighted sum of value of H
  repeat n times:
    $weight := 1$ # weight of current sample
    for each variable X_i in order:
      if $X_i = o_i$ is observed
        $weight := weight \times P(X_i = o_i \mid parents(X_i))$
      else assign X_i a random sample of $P(X_i \mid parents(X_i))$
    $mass := mass + weight$
    $hmass := hmass + weight * $(value of H in current assignment)
  return $hmass/mass$

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                weight := weight × P(X_i = o_i | parents(X_i))
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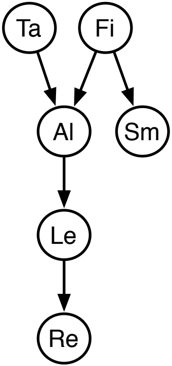

	Ta	Fi	Al	Le	Weight
s_1	true	false	true	false	
s_2	false	true	false	false	
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...					
s_{1000}	false	false	true	true	

$P(sm \mid fi) = 0.9$
$P(sm \mid \neg fi) = 0.01$
$P(re \mid le) = 0.75$
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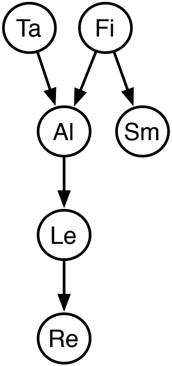

	Ta	Fi	Al	Le	Weight
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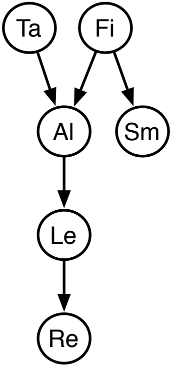

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...					
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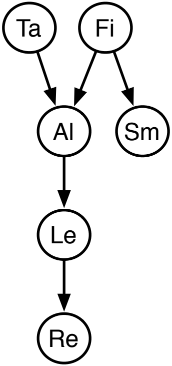
Importance Sampling Example: $P(le \mid sm, ta, \neg re)$



$$
\begin{aligned}
P(ta) &= 0.02 \\
P(fi) &= 0.01 \\
P(al \mid fi \wedge ta) &= 0.5 \\
P(al \mid fi \wedge \neg ta) &= 0.99 \\
P(al \mid \neg fi \wedge ta) &= 0.85 \\
P(al \mid \neg fi \wedge \neg ta) &= 0.0001 \\
P(sm \mid fi) &= 0.9 \\
P(sm \mid \neg fi) &= 0.01 \\
P(le \mid al) &= 0.88 \\
P(le \mid \neg al) &= 0.001 \\
P(re \mid le) &= 0.75 \\
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\end{aligned}
$$

Computing Expectations & Proposal Distributions

Expected value of f with respect to distribution P:

$$\mathbb{E}_P(f) = \sum_w f(w) * P(w)$$

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s is selected according the distribution Q.
The distribution Q is called a proposal distribution.
$P(c) > 0$ then $Q(c) > 0$.
Try to make Q so the weights end up far from zero.

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Importance sampling can be seen as:

for each particle:
    for each variable:
        sample / absorb evidence / update query

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- It works with infinitely many variables (e.g., HMM)
- We can have a new operation of resampling

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- Start with random chosen particles (say 1000)
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  - Absorb evidence: weight each particle by the probability of the evidence observation given the state of the particle.
  - Resample: select each particle at random, in proportion to the weight of the particle.
    Some particles may be duplicated, some may be removed. All new particles have same weight.

Particle Filtering for HMMs

- Start with random chosen particles (say 1000)
- Sample initial states in proportion to their probability.
- Repeat (as each observation arrives):
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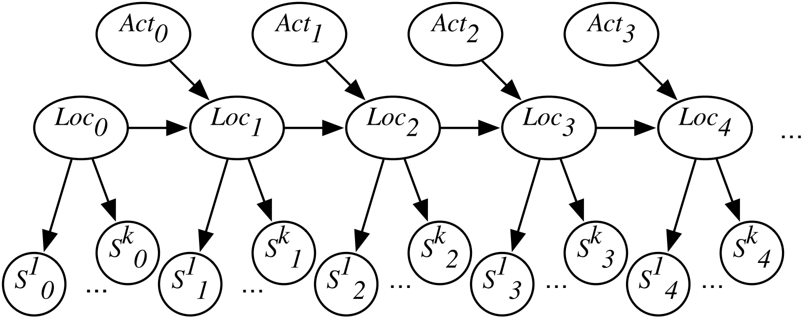
To answer a query about the current state, use the set of particles as data.

Example: Localization



Loc consists of (x, y, θ) – position and orientation

$k = 24$ sonar sensors (all very noisy)

Markov Chain Monte Carlo

To sample from a distribution P:

- Create (ergodic and aperiodic) Markov chain with P as equilibrium distribution.
  Let $T(S_{i+1} \mid S_i)$ be the transition probability.
- Given state s, sample state s' from $T(S \mid s)$

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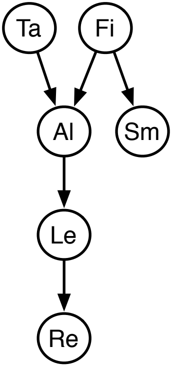
- Samples are not independent of each other "autocorrelation".
  Sometimes use subset (e.g., 1/1000) of them "thinning"
- Gibbs sampler: sample each non-observed variable from the distribution of the variable given the current (or observed) value of the variables in its Markov blanket.

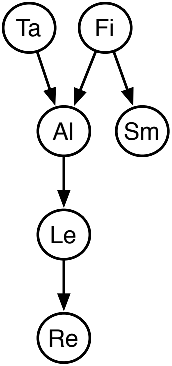
Gibbs Sampling Example: $P(ta \mid sm, re)$



	Ta	Fi	Al	Le
s_1	true	false	false	true

Select *Le*.

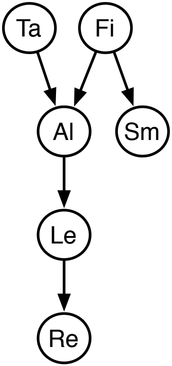
Gibbs Sampling Example: $P(ta \mid sm, re)$



	Ta	Fi	Al	Le
s_1	true	false	false	true

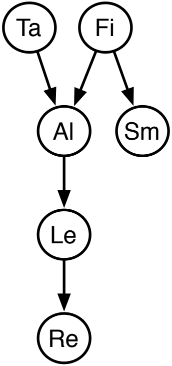
Select Le. Sample from $P(Le \mid \neg al \wedge re)$

Gibbs Sampling Example: $P(ta \mid sm, re)$



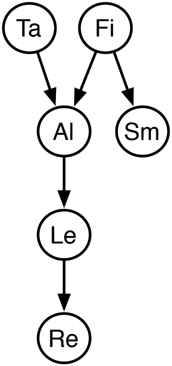
	Ta	Fi	Al	Le
s_1	true	false	false	true
Select *Le*. Sample from $P(Le \mid \neg al \wedge re)$				
s_2	true	false	false	false

Gibbs Sampling Example: $P(ta \mid sm, re)$



	Ta	Fi	Al	Le
s_1	true	false	false	true
Select Le. Sample from $P(Le \mid \neg al \wedge re)$				
s_2	true	false	false	false
Select Fi.				

Gibbs Sampling Example: $P(ta \mid sm, re)$



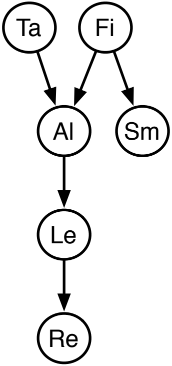
	Ta	Fi	Al	Le
s_1	true	false	false	true

Select Le. Sample from $P(Le \mid \neg al \wedge re)$

	Ta	Fi	Al	Le
s_2	true	false	false	false

Select Fi. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$

Gibbs Sampling Example: $P(ta \mid sm, re)$



	Ta	Fi	Al	Le
s_1	true	false	false	true
Select *Le*. Sample from $P(Le \mid \neg al \wedge re)$				
s_2	true	false	false	false
Select *Fi*. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$				
s_3	true	true	false	false

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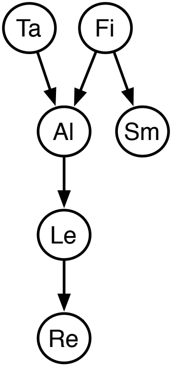


	Ta	Fi	Al	Le
s_1	true	false	false	true

Select Le. Sample from $P(Le \mid \neg al \wedge re)$

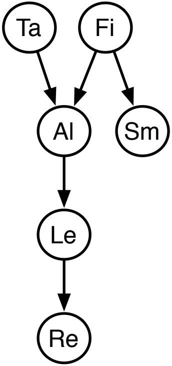
s_2	true	false	false	false

Select Fi. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$

s_3	true	true	false	false

Select Al.

Gibbs Sampling Example: $P(ta \mid sm, re)$



	Ta	Fi	Al	Le
s_1	true	false	false	true

Select *Le*. Sample from $P(Le \mid \neg al \wedge re)$

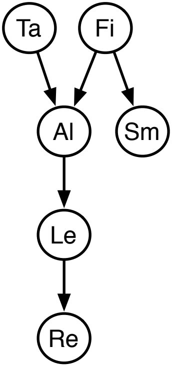
	Ta	Fi	Al	Le
s_2	true	false	false	false

Select *Fi*. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$

	Ta	Fi	Al	Le
s_3	true	true	false	false

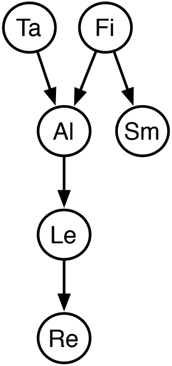
Select *Al*. Sample from $P(Al \mid ta \wedge fi \wedge \neg le)$

Gibbs Sampling Example: $P(ta \mid sm, re)$



	Ta	Fi	Al	Le
s_1	true	false	false	true
Select Le. Sample from $P(Le \mid \neg al \wedge re)$				
s_2	true	false	false	false
Select Fi. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$				
s_3	true	true	false	false
Select Al. Sample from $P(Al \mid ta \wedge fi \wedge \neg le)$				
s_4	true	true	false	false

Gibbs Sampling Example: $P(ta \mid sm, re)$



	Ta	Fi	Al	Le
s_1	true	false	false	true
Select *Le*. Sample from $P(Le \mid \neg al \wedge re)$				
s_2	true	false	false	false
Select *Fi*. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$				
s_3	true	true	false	false
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s_4	true	true	false	false
Select *Le*.				

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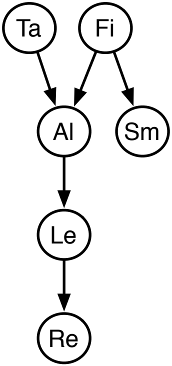

	Ta	Fi	Al	Le
s_1	true	false	false	true

Select Le. Sample from $P(Le \mid \neg al \wedge re)$

	Ta	Fi	Al	Le
s_2	true	false	false	false

Select Fi. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$

	Ta	Fi	Al	Le
s_3	true	true	false	false

Select Al. Sample from $P(Al \mid ta \wedge fi \wedge \neg le)$

	Ta	Fi	Al	Le
s_4	true	true	false	false

Select Le. Sample from $P(Le \mid \neg al \wedge re)$

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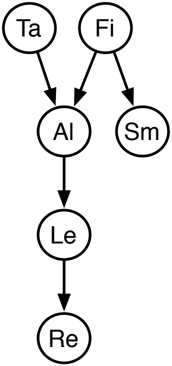

	Ta	Fi	Al	Le
s_1	true	false	false	true
Select Le. Sample from $P(Le \mid \neg al \wedge re)$				
s_2	true	false	false	false
Select Fi. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$				
s_3	true	true	false	false
Select Al. Sample from $P(Al \mid ta \wedge fi \wedge \neg le)$				
s_4	true	true	false	false
Select Le. Sample from $P(Le \mid \neg al \wedge re)$				
s_5	true	true	false	true

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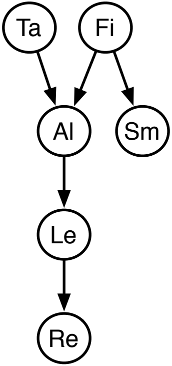

	Ta	Fi	Al	Le
s_1	true	false	false	true
Select Le. Sample from $P(Le \mid \neg al \wedge re)$				
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Select Fi. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$				
s_3	true	true	false	false
Select Al. Sample from $P(Al \mid ta \wedge fi \wedge \neg le)$				
s_4	true	true	false	false
Select Le. Sample from $P(Le \mid \neg al \wedge re)$				
s_5	true	true	false	true
Select Ta.				

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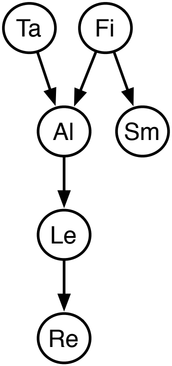

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s_1	true	false	false	true
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Select Fi. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$				
s_3	true	true	false	false
Select Al. Sample from $P(Al \mid ta \wedge fi \wedge \neg le)$				
s_4	true	true	false	false
Select Le. Sample from $P(Le \mid \neg al \wedge re)$				
s_5	true	true	false	true
Select Ta. Sample from $P(Ta \mid \neg al \wedge fi)$				

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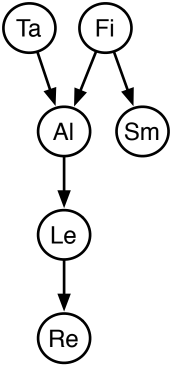

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s_3	true	true	false	false
Select Al. Sample from $P(Al \mid ta \wedge fi \wedge \neg le)$				
s_4	true	true	false	false
Select Le. Sample from $P(Le \mid \neg al \wedge re)$				
s_5	true	true	false	true
Select Ta. Sample from $P(Ta \mid \neg al \wedge fi)$				
s_6	true	true	false	true

...