Stochastic Simulation

- Idea: probabilities $\leftrightarrow$ samples
- Get probabilities from samples:

X	$count$
x_1	n_1
$\vdots$	$\vdots$
x_k	n_k
$total$	m

$\leftrightarrow$

X	$probability$
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- If we could sample from a variable's (posterior) probability, we could estimate its (posterior) probability.

Generating samples from a distribution

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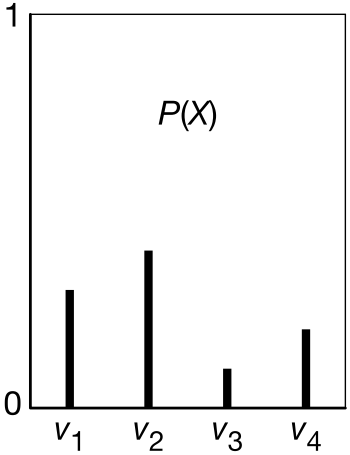
- Select a value y uniformly in the range $[0, 1]$.
- Select the x such that $f(x) = y$.

Cumulative Distribution



Hoeffding's inequality

Theorem (Hoeffding): Suppose p is the true probability, and s is the sample average from n independent samples; then

$$P(|s - p| > \epsilon) \leq 2e^{-2n\epsilon^2}.$$

Guarantees a probably approximately correct estimate of probability.

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ϵ	δ	n
0.1	0.05	185
0.01	0.05	18,445
0.1	0.01	265

Forward sampling in a belief network

- Sample the variables one at a time; sample parents of X before sampling X.
- Given values for the parents of X, sample from the probability of X given its parents.

Rejection Sampling

- To estimate a posterior probability given evidence $Y_1 = v_1 \wedge \ldots \wedge Y_j = v_j$:
- Reject any sample that assigns Y_i to a value other than v_i.

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- Reject any sample that assigns Y_i to a value other than v_i.
- The non-rejected samples are distributed according to the posterior probability:

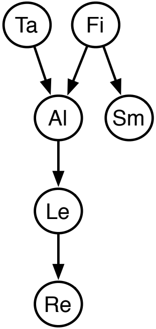
$$P(\alpha \mid evidence) \approx \frac{\sum_{\alpha} \text{is true in sample } 1}{\sum_{sample} 1}$$

where we consider only samples consistent with evidence.

Rejection Sampling Example: $P(ta \mid sm, re)$

Observe $Sm = true, Re = true$

	Ta	Fi	Al	Sm	Le	Re
s_1	false	true	false	true	false	false



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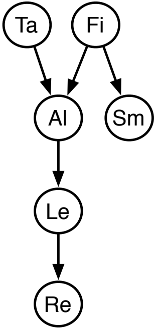

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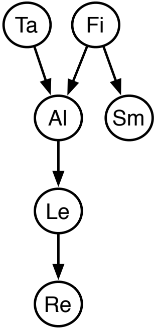

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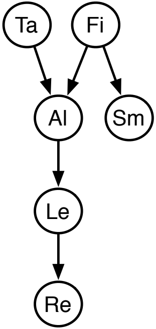

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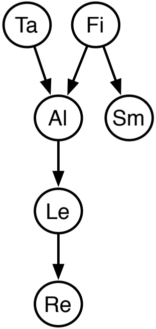

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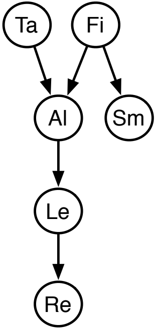

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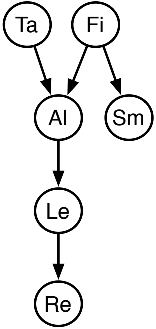

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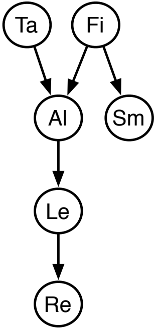

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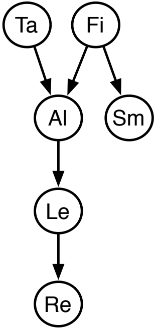

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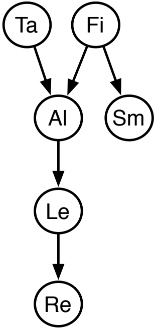

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$P(sm) = 0.02$
$P(re \mid sm) = 0.32$
There are 1000 samples.

How many are rejected?

How many are used?

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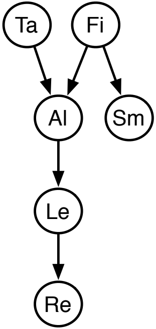

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Doesn't work well when evidence is unlikely.

Importance Sampling

- Samples have weights: a real number associated with each sample that takes the evidence into account.

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- Mix exact inference with sampling: don't sample all of the variables, but weight each sample according to $P(\mathit{evidence} \mid \mathit{sample})$.

Importance Sampling (Likelihood Weighting)

procedure *likelihood_weighting*(Bn, e, H, n):
  \# Approximate $P(H \mid e)$ in belief network Bn using n samples.
  \# H has some real domain (e.g., $\{0, 1\}$)
  $mass := 0$ \# mass of all samples
  $hmass := 0$ \# weighted sum of value of H
  repeat n times:
    $weight := 1$ \# weight of current sample
    for each variable X_i in order:
      if $X_i = o_i$ is observed
        $weight := weight \times P(X_i = o_i \mid parents(X_i))$
      else assign X_i a random sample of $P(X_i \mid parents(X_i))$
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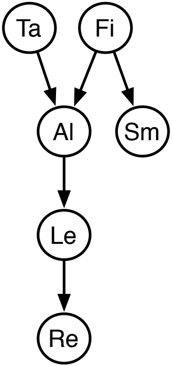

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s_1	true	false	true	false	
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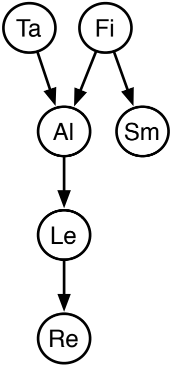

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s_1	true	false	true	false	0.01×0.01
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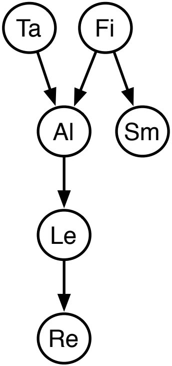

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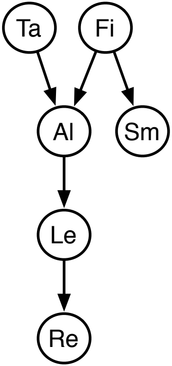
Importance Sampling Example: $P(le \mid sm, ta, \neg re)$



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$P(al \mid fi \wedge ta) = 0.5$
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The distribution Q is called a proposal distribution.
$P(c) > 0$ then $Q(c) > 0$.
Try to make Q so the weights end up far from zero.

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for each particle:

for each variable:

sample / absorb evidence / update query

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- We can have a new operation of resampling

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- Start with random chosen particles (say 1000)
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  - Transition: sample the next state for each particle according to the transition probabilities.

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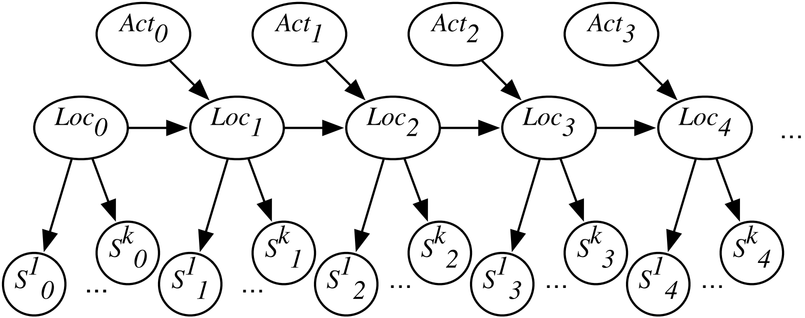
- Sample initial states in proportion to their probability.
- Repeat (as each observation arrives):
  - Absorb evidence: weight each particle by the probability of the evidence observation given the state of the particle.
  - Resample: select each particle at random, in proportion to the weight of the particle.
    Some particles may be duplicated, some may be removed. All new particles have same weight.
  - Transition: sample the next state for each particle according to the transition probabilities.

To answer a query about the current state, use the set of particles as data.

Example: Localization



Loc consists of (x, y, θ) – position and orientation

$k = 24$ sonar sensors (all very noisy)

Markov Chain Monte Carlo

To sample from a distribution P:

- Create (ergodic and aperiodic) Markov chain with P as equilibrium distribution.
  Let $T(S_{i+1} \mid S_i)$ be the transition probability.
- Given state s, sample state s' from $T(S \mid s)$

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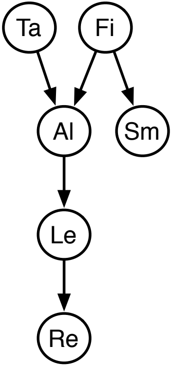
- Samples are not independent of each other "autocorrelation".
  Sometimes use subset (e.g., 1/1000) of them "thinning"
- Gibbs sampler: sample each non-observed variable from the distribution of the variable given the current (or observed) value of the variables in its Markov blanket.

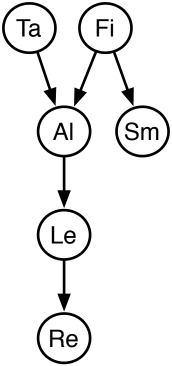
Gibbs Sampling Example: $P(ta \mid sm, re)$



	Ta	Fi	Al	Le
s_1	true	false	false	true

Select Le.

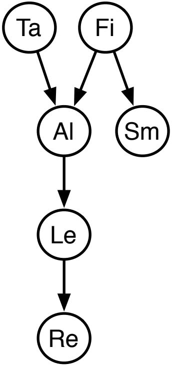
Gibbs Sampling Example: $P(ta \mid sm, re)$



	Ta	Fi	Al	Le
s_1	true	false	false	true

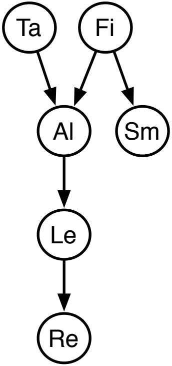
Select *Le*. Sample from $P(Le \mid \neg al \wedge re)$

Gibbs Sampling Example: $P(ta \mid sm, re)$



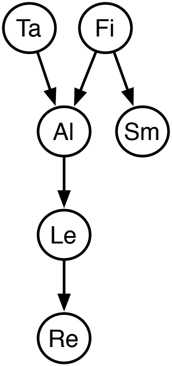
	Ta	Fi	Al	Le
s_1	true	false	false	true
Select *Le*. Sample from $P(Le \mid \neg al \wedge re)$				
s_2	true	false	false	false

Gibbs Sampling Example: $P(ta \mid sm, re)$



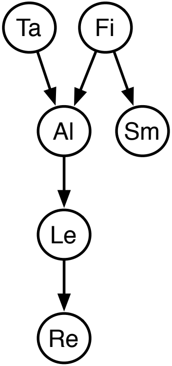
	Ta	Fi	Al	Le
s_1	true	false	false	true
Select Le. Sample from $P(Le \mid \neg al \wedge re)$				
s_2	true	false	false	false
Select Fi.				

Gibbs Sampling Example: $P(ta \mid sm, re)$



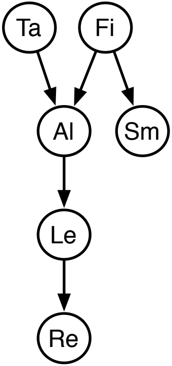
	Ta	Fi	Al	Le
s_1	true	false	false	true
Select Le. Sample from $P(Le \mid \neg al \wedge re)$				
s_2	true	false	false	false
Select Fi. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$				

Gibbs Sampling Example: $P(ta \mid sm, re)$



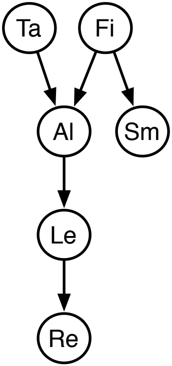
	Ta	Fi	Al	Le
s_1	true	false	false	true
Select Le. Sample from $P(Le \mid \neg al \wedge re)$				
s_2	true	false	false	false
Select Fi. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$				
s_3	true	true	false	false

Gibbs Sampling Example: $P(ta \mid sm, re)$



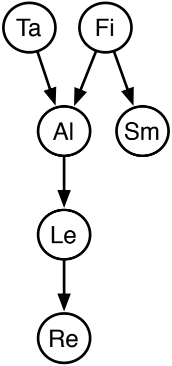
	Ta	Fi	Al	Le
s_1	true	false	false	true
Select *Le*. Sample from $P(Le \mid \neg al \wedge re)$				
s_2	true	false	false	false
Select *Fi*. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$				
s_3	true	true	false	false
Select *Al*.				

Gibbs Sampling Example: $P(ta \mid sm, re)$



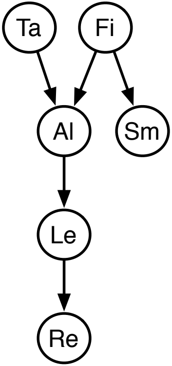
	Ta	Fi	Al	Le
s_1	true	false	false	true
Select Le. Sample from $P(Le \mid \neg al \wedge re)$				
s_2	true	false	false	false
Select Fi. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$				
s_3	true	true	false	false
Select Al. Sample from $P(Al \mid ta \wedge fi \wedge \neg le)$				

Gibbs Sampling Example: $P(ta \mid sm, re)$



	Ta	Fi	Al	Le
s_1	true	false	false	true
Select Le. Sample from $P(Le \mid \neg al \wedge re)$				
s_2	true	false	false	false
Select Fi. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$				
s_3	true	true	false	false
Select Al. Sample from $P(Al \mid ta \wedge fi \wedge \neg le)$				
s_4	true	true	false	false

Gibbs Sampling Example: $P(ta \mid sm, re)$



	Ta	Fi	Al	Le
s_1	true	false	false	true

Select Le. Sample from $P(Le \mid \neg al \wedge re)$

	Ta	Fi	Al	Le
s_2	true	false	false	false

Select Fi. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$

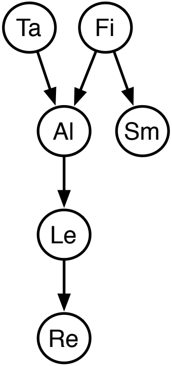
	Ta	Fi	Al	Le
s_3	true	true	false	false

Select Al. Sample from $P(Al \mid ta \wedge fi \wedge \neg le)$

	Ta	Fi	Al	Le
s_4	true	true	false	false

Select Le.

Gibbs Sampling Example: $P(ta \mid sm, re)$



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s_3	true	true	false	false
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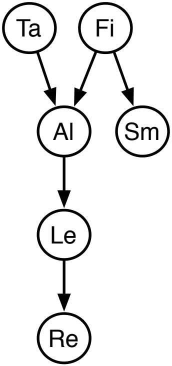
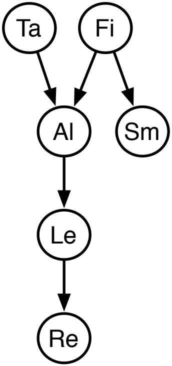


	Ta	Fi	Al	Le
s_1	true	false	false	true
Select Le. Sample from $P(Le \mid \neg al \wedge re)$				
s_2	true	false	false	false
Select Fi. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$				
s_3	true	true	false	false
Select Al. Sample from $P(Al \mid ta \wedge fi \wedge \neg le)$				
s_4	true	true	false	false
Select Le. Sample from $P(Le \mid \neg al \wedge re)$				
s_5	true	true	false	true

Gibbs Sampling Example: $P(ta \mid sm, re)$



	Ta	Fi	Al	Le
s_1	true	false	false	true

Select Le. Sample from $P(Le \mid \neg al \wedge re)$

s_2	true	false	false	false

Select Fi. Sample from $P(Fi \mid ta \wedge \neg al \wedge sm)$

s_3	true	true	false	false

Select Al. Sample from $P(Al \mid ta \wedge fi \wedge \neg le)$

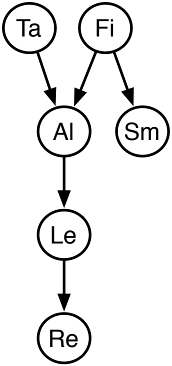
s_4	true	true	false	false

Select Le. Sample from $P(Le \mid \neg al \wedge re)$

s_5	true	true	false	true

Select Ta.

Gibbs Sampling Example: $P(ta \mid sm, re)$



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s_3	true	true	false	false
Select Al. Sample from $P(Al \mid ta \wedge fi \wedge \neg le)$				
s_4	true	true	false	false
Select Le. Sample from $P(Le \mid \neg al \wedge re)$				
s_5	true	true	false	true
Select Ta. Sample from $P(Ta \mid \neg al \wedge fi)$				

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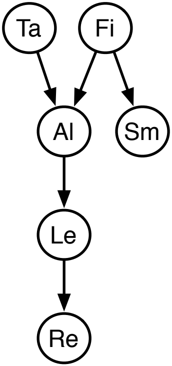

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s_3	true	true	false	false
Select *Al*. Sample from $P(Al \mid ta \wedge fi \wedge \neg le)$				
s_4	true	true	false	false
Select *Le*. Sample from $P(Le \mid \neg al \wedge re)$				
s_5	true	true	false	true
Select *Ta*. Sample from $P(Ta \mid \neg al \wedge fi)$				
s_6	true	true	false	true

...