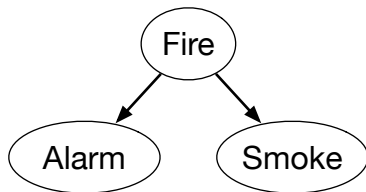


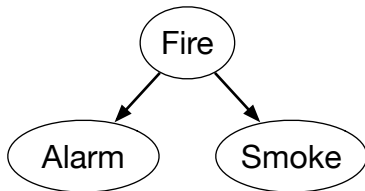
# Understanding Independence: Common ancestors

- *Alarm* and *Smoke* are given  $\{\}$



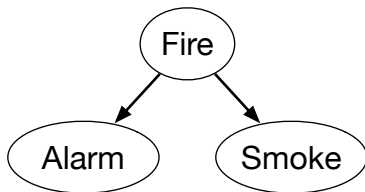
# Understanding Independence: Common ancestors

- *Alarm* and *Smoke* are dependent given  $\{\}$



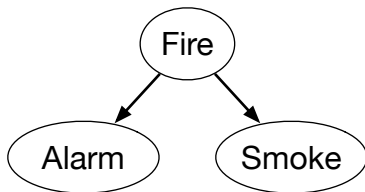
# Understanding Independence: Common ancestors

- *Alarm* and *Smoke* are dependent given  $\{\}$
- *Alarm* and *Smoke* are given *Fire*

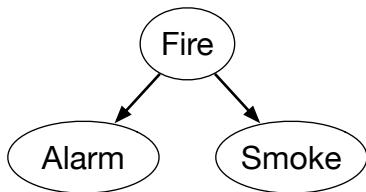


# Understanding Independence: Common ancestors

- *Alarm* and *Smoke* are dependent given  $\{\}$
- *Alarm* and *Smoke* are independent given *Fire*

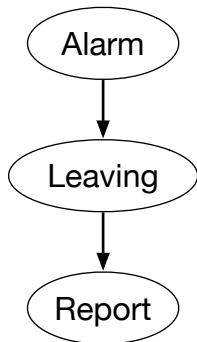


# Understanding Independence: Common ancestors

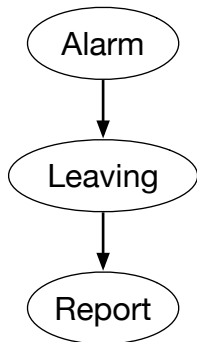


- *Alarm* and *Smoke* are dependent given  $\{\}$
- *Alarm* and *Smoke* are independent given *Fire*
- Intuitively, *Fire* can **explain** *Alarm* and *Smoke*; learning one can affect the other by changing the belief in *Fire*.

# Understanding Independence: Chain

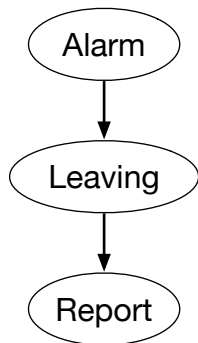


- *Alarm* and *Report* are given  $\{\}$



- *Alarm* and *Report* are dependent given  $\{\}$

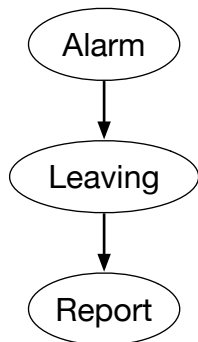
# Understanding Independence: Chain



- *Alarm* and *Report* are dependent given  $\{\}$
- *Alarm* and *Report* are given *Leaving*

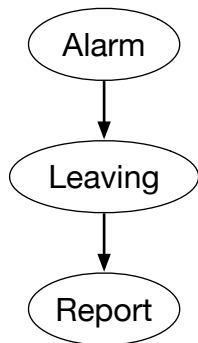


# Understanding Independence: Chain



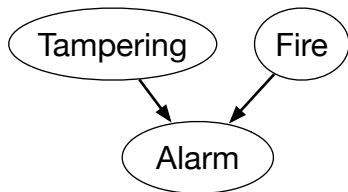
- *Alarm* and *Report* are dependent given  $\{\}$
- *Alarm* and *Report* are independent given *Leaving*

# Understanding Independence: Chain



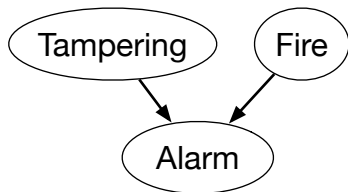
- *Alarm* and *Report* are dependent given  $\{\}$
- *Alarm* and *Report* are independent given *Leaving*
- The (only) way that the *Alarm* affects *Report* is by affecting *Leaving*.

# Understanding Independence: Common descendants



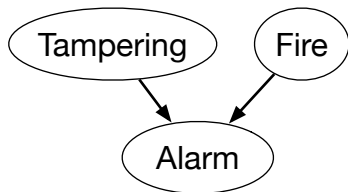
- *Tampering* and *Fire* are given  $\{\}$

# Understanding Independence: Common descendants



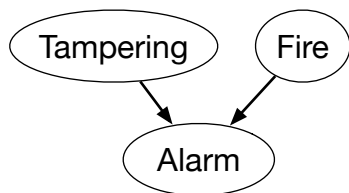
- *Tampering* and *Fire* are independent given  $\{\}$

# Understanding Independence: Common descendants



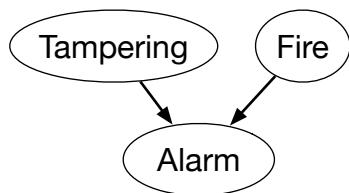
- *Tampering* and *Fire* are independent given  $\{\}$
- *Tampering* and *Fire* are given *Alarm*

# Understanding Independence: Common descendants



- *Tampering* and *Fire* are independent given  $\{\}$
- *Tampering* and *Fire* are dependent given *Alarm*

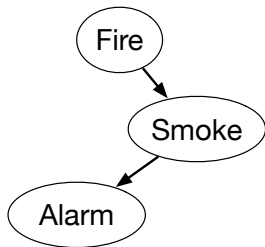
# Understanding Independence: Common descendants



- *Tampering* and *Fire* are independent given  $\{\}$
- *Tampering* and *Fire* are dependent given *Alarm*
- Intuitively, *Tampering* can **explain away** *Fire*

# Clicker Question

For the belief network, and the ordering *Fire, Smoke, Alarm*

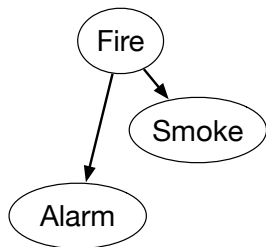


- A *Alarm* is independent of *Smoke* given *Fire*
- B *Alarm* is independent of *Fire* given *Smoke*
- C *Alarm* is independent of *Fire* given  $\{\}$
- D All of the above independencies hold
- E There are no independencies



# Clicker Question

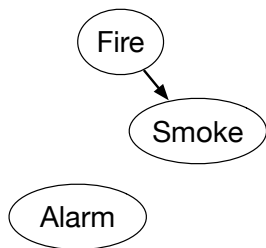
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## Clicker Question

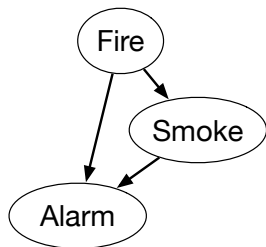
For the belief network, and the ordering *Fire, Smoke, Alarm*



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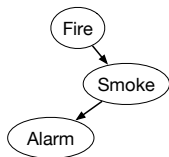
For the belief network, and the ordering *Fire, Smoke, Alarm*



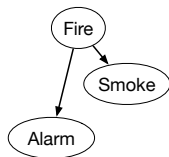
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# Clicker Question

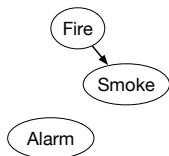
Which network best fits a fire alarm that only detects the heat of the fire?



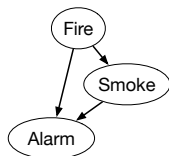
A



B



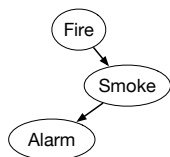
C



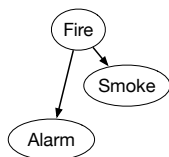
D

# Clicker Question

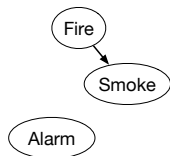
Which network best fits a smoke alarm (that only detects smoke)?



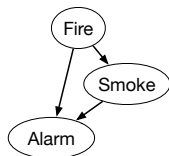
A



B



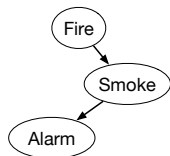
C



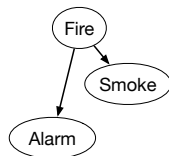
D

# Clicker Question

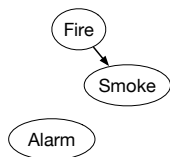
Which network best fits a fire alarm that detects both smoke and the heat of the fire?



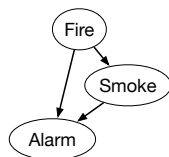
A



B



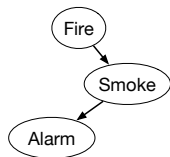
C



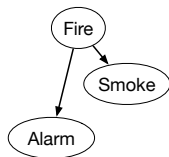
D

# Clicker Question

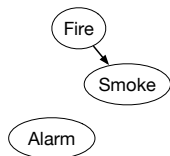
Which network best fits a burglary alarm that doesn't detect heat or smoke?



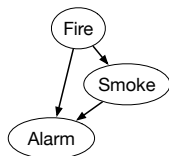
A



B

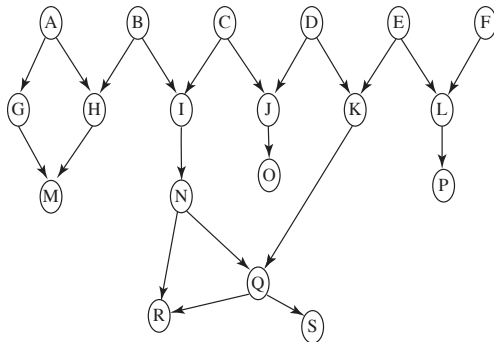


C



D

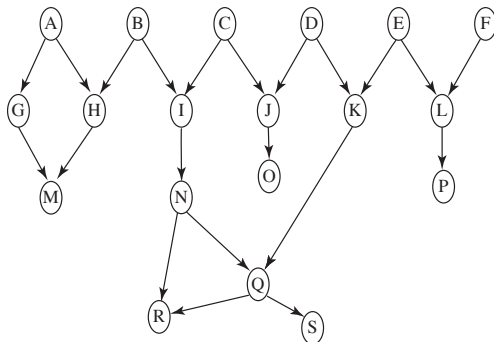
# Understanding independence: example



(a) On which given probabilities does  $P(N)$  depend?

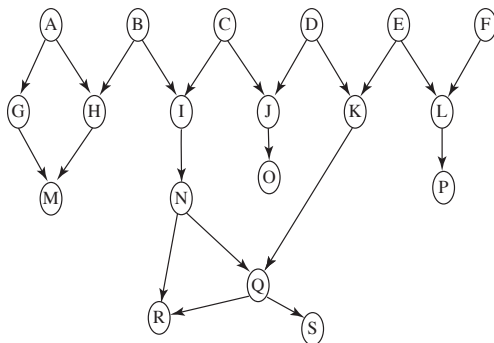


# Understanding independence: example



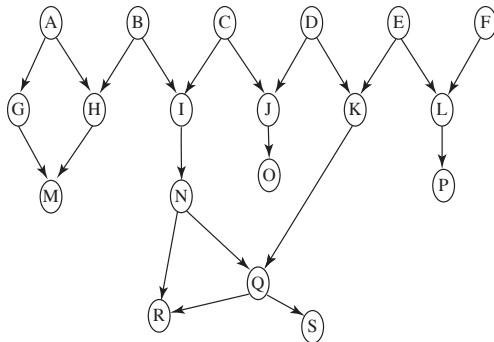
- (a) On which given probabilities does  $P(N)$  depend?
- (b) If you were to observe a value for  $B$ , which variables' probabilities will change?

# Understanding independence: example



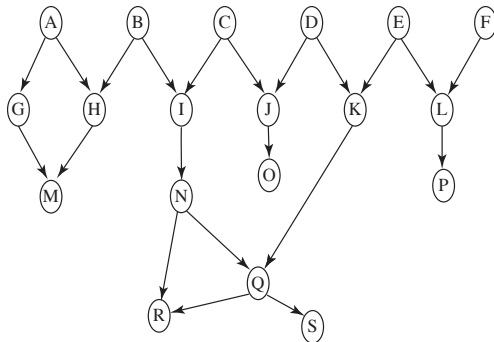
- (a) On which given probabilities does  $P(N)$  depend?
- (b) If you were to observe a value for  $B$ , which variables' probabilities will change?
- (c) If you were to observe a value for  $N$ , which variables' probabilities will change?

# Understanding independence: questions



- (d) Suppose you had observed a value for  $M$ ; if you were to then observe a value for  $N$ , which variables' probabilities will change?

# Understanding independence: questions



- (d) Suppose you had observed a value for  $M$ ; if you were to then observe a value for  $N$ , which variables' probabilities will change?
- (e) Suppose you had observed  $B$  and  $Q$ ; which variables' probabilities will change when you observe  $N$ ?

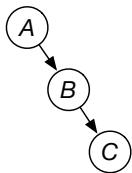
# What variables are affected by observing?

- If you observe variable(s)  $\bar{Y}$ , the variables whose posterior probability is different from their prior are:
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  - ▶ their descendants.

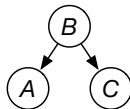
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- If you observe variable(s)  $\bar{Y}$ , the variables whose posterior probability is different from their prior are:
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  - ▶ their descendants.
- Intuitively (assuming network ordered so causes are before effects):
  - ▶ You do **abduction** to possible causes and
  - ▶ **prediction** from the causes.

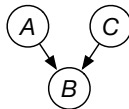
# Three types of meetings between arcs



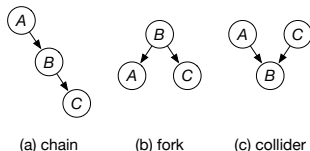
(a) chain



(b) fork



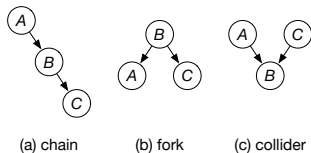
(c) collider



- A **path**  $p$  can follow arrows in either direction.
- Observations  $Zs$  **block** a path  $p$  if:
  - (a)  $p$  contains a **chain**  $A \rightarrow B \rightarrow C$ , and  $B \in Zs$
  - (b)  $p$  contains a **fork**  $A \leftarrow B \rightarrow C$ , and  $B \in Zs$
  - (c)  $p$  contains a **collider**  $A \rightarrow B \leftarrow C$ , and  $B$ , and all its descendants, are **not** in  $Zs$

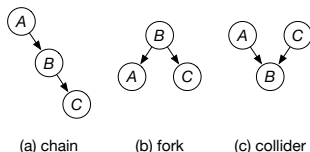


# D-separation



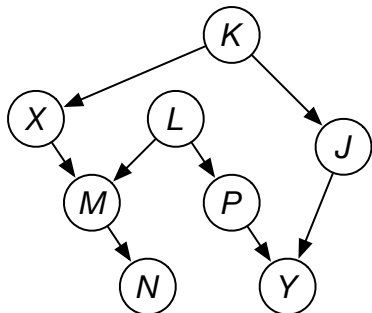
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- $X$  is **d-separated** from  $Y$  given  $Zs$  if every path between  $X$  and  $Y$  is blocked by  $Zs$

# D-separation



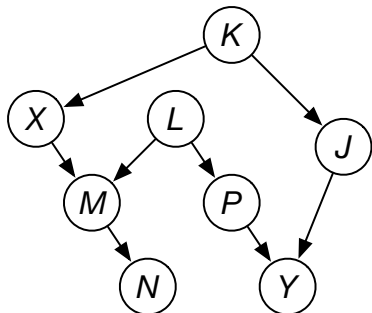
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- $X$  is **d-separated** from  $Y$  given  $Zs$  if every path between  $X$  and  $Y$  is blocked by  $Zs$
- $X$  is independent  $Y$  given  $Zs$  for all conditional probabilities iff  $X$  is d-separated from  $Y$  given  $Zs$

# Example



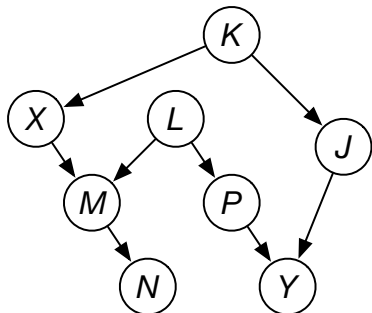
- Are  $X$  and  $Y$  d-separated by  $\{ \}$ ?

# Example



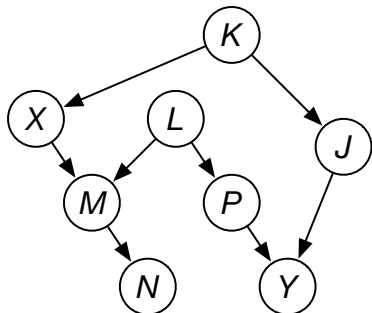
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## Example



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## Example



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- Are  $X$  and  $Y$  d-separated by  $\{K, N, P\}$ ?

# Markov Random Field

A **Markov random field** is composed of

- of a set of random variables:  $\mathbf{X} = \{X_1, X_2, \dots, X_n\}$  and
- a set of factors  $\{f_1, \dots, f_m\}$ , where a factor is a non-negative function of a subset of the variables.

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$$P(\mathbf{X} = \mathbf{x}) = \frac{1}{Z} \prod_k f_k(\mathbf{X}_k = \mathbf{x}_k) .$$

$$Z = \sum_{\mathbf{x}} \prod_k f_k(\mathbf{X}_k = \mathbf{x}_k)$$

where  $f_k(\mathbf{X}_k)$  is a factor on  $\mathbf{X}_k \subseteq \mathbf{X}$ , and  $\mathbf{x}_k$  is  $\mathbf{x}$  projected onto  $\mathbf{X}_k$ .

$Z$  is a normalization constant known as the **partition function**.

# Markov Networks and Factor graphs

- A **factor graph** is a bipartite graph, which contains a variable node for each random variable and a factor node for each factor. There is an edge between a variable node and a factor node if the variable appears in the factor.

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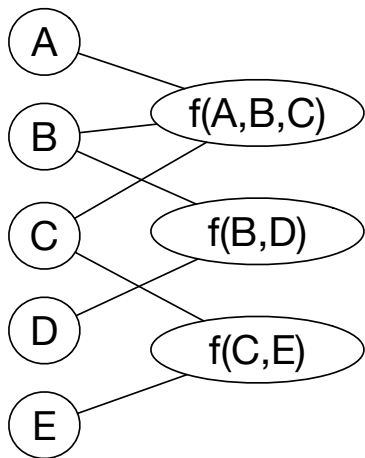
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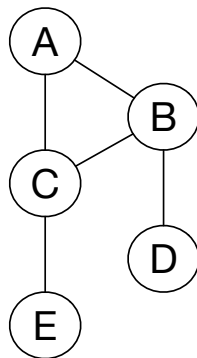
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- A **belief network** is a type of Markov random field where the factors represent conditional probabilities, there is a factor for each variable, and directed graph is acyclic.

# Factor graph and Markov network example



Factor Graph



Markov Network

# Independence in a Markov Network

- The **Markov blanket** of a variable  $X$  is the set of variables that are in factors with  $X$ .
- A variable is independent of the other variables given its Markov blanket.

# Independence in a Markov Network

- The **Markov blanket** of a variable  $X$  is the set of variables that are in factors with  $X$ .
- A variable is independent of the other variables given its Markov blanket.
- $X$  is **connected** to  $Y$  given  $\bar{Z}$  if there is a path from  $X$  to  $Y$  in the Markov network, which does not contain an element of  $Z$ .
- $X$  is **separated** from  $Y$  given  $\bar{Z}$  if it is not connected.



# Independence in a Markov Network

- The **Markov blanket** of a variable  $X$  is the set of variables that are in factors with  $X$ .
- A variable is independent of the other variables given its Markov blanket.
- $X$  is **connected** to  $Y$  given  $\bar{Z}$  if there is a path from  $X$  to  $Y$  in the Markov network, which does not contain an element of  $Z$ .
- $X$  is **separated** from  $Y$  given  $\bar{Z}$  if it is not connected.
- A **positive** factor is one that does not contain zero values.
- $\bar{X}$  is independent  $\bar{Y}$  given  $\bar{Z}$  for all positive factors iff  $\bar{X}$  is separated from  $\bar{Y}$  given  $\bar{Z}$

# Canonical Representations

- The **parameters** of a graphical model are the numbers that define the model.
- A belief network is a **canonical representation**: given the structure and the distribution, the parameters are uniquely determined.
- A Markov random field is not a canonical representation. Many different parameterizations result in the same distribution.