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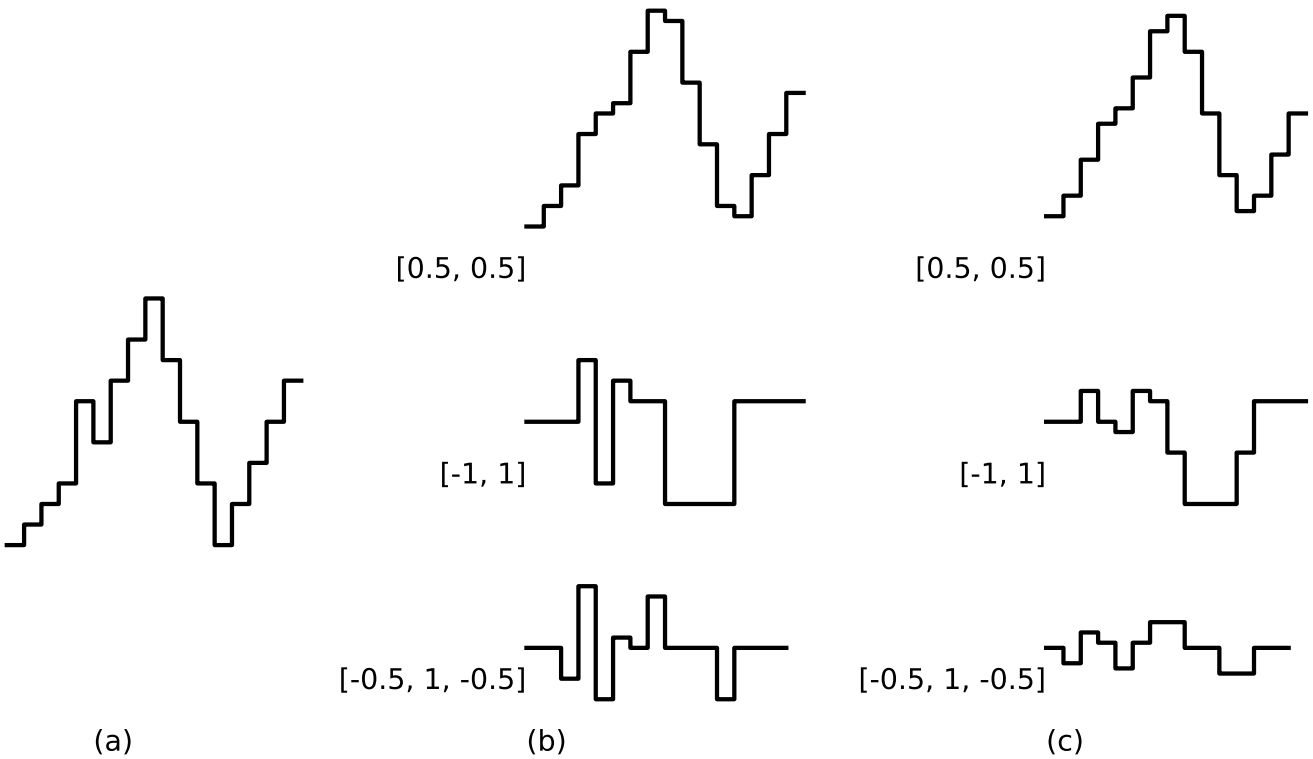
  How well would humans perform with this new data set?
- If it had learned to recognize cats in the top of images, how does it perform with cats in the bottom of images?
- Convolutional neural networks tackle these problems by using filters that act on small patches of an image, and by sharing the parameters so they learn useful features no matter where in an image they occur.

Kernels (one-dimensional case)

- A kernel (convolution mask, or filter) is a learned linear operator that is applied to local patches.
- In one dimension, suppose the input is a sequence (list) $[x_0, \ldots, x_{m-1}]$ and there is structure so that x_i is close to x_{i+1} in some sense.
- A one-dimensional kernel is a vector $[w_0, \ldots, w_{k-1}]$, where k is the kernel size, which when applied to the sequence $x = [x_0, \ldots, x_{m-1}]$ produces a sequence $y = [y_0, \ldots, y_{m-k}]$ where

$$y[i] = \sum_{j=0}^{k-1} x[i+j] * w[j].$$

One-dimensional kernels example



(a) original signal $[0, 1, 2, 3, 7, 5, 8, 10, 12, 9, 6, 3, 0, 2, 4, 6, 8]$

(b) kernels $[0.5, 0.5]$, $[-1, 1]$, and $[-0.5, 1, -0.5]$ applied to the signal (a)

(c) same kernels applied to the top signal in (b)

Two-dimensional kernels

- A two-dimensional kernel is a $j \times k$ array.
- This kernel $w[i, j]$ applied to the two-dimensional array *in* produces a two-dimensional array *out* where

$$out[x, y] := \sum_{i=0}^{j-1} \sum_{j=0}^{k-1} in[x + i, y + j] * w[i, j].$$

  The kernel size, $j \times k$, is usually much smaller than the size of the dimension of *in*.
- Two-dimensional kernels are used for images, where they are applied to patches of adjacent pixels.

Two-dimensional kernel examples

What do the following kernels applied to a black-and-white image, where $in[x, y]$ is the brightness of the pixel at position (x, y) do?

0	1
−1	0

(a)

1	0
0	−1

(b)

- Kernels (a) and (b) were invented by Roberts [1965].

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- Kernels (a) and (b) were invented by Roberts [1965].
- Kernels (c) and (d) are the Sobel–Feldman operators
- Now kernels are mostly learned.

Convolutional Neural Networks

Convolutional neural networks learn kernels weights from data. Two main aspects distinguish convolutional neural networks from ordinary neural networks:

- Locality: the values are a function of neighboring positions, rather than being based on all units as they are in a fully-connected layer.
- Parameter sharing or weight tying: the same parameters in a kernel are used at all locations in an image.

Conv2D (for $k \times k$ kernel)

class Conv2D(k)
    initialize $w[i,j]$ randomly, and $d[i,j]$ to 0
    def *output*(*input*) ▷ *input* is $xd \times yd$ array
        for each $x : 0 \leq x \leq xd - k$, $y : 0 \leq y \leq yd - k$ **do**

$$out[x,y] := \sum_{i=0}^{k-1} \sum_{j=0}^{k-1} in[x+i, y+j] * w[i,j]$$

        return *out*
    def *Backprop*(*error*)
        Initialize $ierror[x,y]$ to 0
        for each $x : 0 \leq x \leq xd - k$, $y : 0 \leq y \leq yd - k$ **do**
            for each $i : 0 \leq i < k$, $j : 0 \leq j < k$ **do**
                $d[i,j]$ += $in[x+i, y+j] * error[x,y]$
                $ierror[x+i, y+j]$ += $error[x,y] * w[i,j]$
        return *ierror*
    def *update*() ▷ Same as for *Dense*

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- Whether to use a bias is a parameter in most libraries.

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- A fully-convolutional network can makes a prediction of each pixel (e.g., where is there a cat)
- A shortcut connection or a skip connection is a connection that skips some layers.
- In a residual network, the output from one layer are added to the outputs from a lower layer. The layer learns to fix errors of lower layers, as in boosting (Section 7.5.1)