

Overfitting

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 - ▶ **overconfidence**: more extreme probabilities than is justified.
- Fitting the training set better does not mean fitting the test set or better predictions of future cases

Example of Overfitting: Restaurant Ratings

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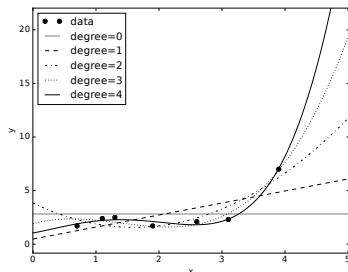
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- Ratings are a mix of quality and luck. Lots of data averages out luck.

Example of Overfitting: Model complexity

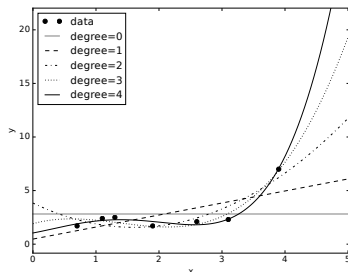
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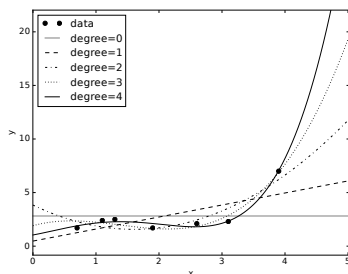
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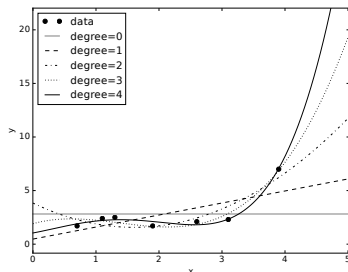
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- What happens with extrapolation?
How do polynomials of odd and even degrees differ?

Test set error is caused by:

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- **Noise:** inherent error due to the data depending on features not modeled or the process generating the data is inherently stochastic.

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 - ▶ Don't need to store pseudo-examples, just the sufficient statistics: pseudocounts.

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A theoretical justification of pseudocounts is given in Chapter 10.

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- **Regularization**: optimize fit-to-data plus a term that penalizes complexity
- Find a predictor \hat{Y} to minimize

$$\left(\sum_e \text{loss}(\hat{Y}(e), Y(e)) \right) + \lambda * \text{regularizer}(\hat{Y})$$

- ▶ $\text{loss}(\hat{Y}(e), Y(e))$ is the loss of example e for predictor \hat{Y}
- ▶ $\text{regularizer}(\hat{Y})$ is a penalty term that penalizes complexity.
- ▶ The **regularization parameter**, λ , trades off fit-to-data and model simplicity

Regularization

- In **decision tree learning**, one complexity measure is the number of leaves in a decision tree.
- When building a decision tree, you could optimize the sum of a loss plus a function of the size of the decision tree, minimizing

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- When greedily splitting, a single split is worthwhile if it reduces the sum of losses by γ .

L1 and L2 Regularization

Linear/logistic regression, minimize sum-of-squares:

$$\text{minimize } Error_E(\bar{w}) = \sum_{e \in E} \left(Y(e) - f\left(\sum_i w_i X_i(e)\right) \right)^2.$$

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λ is a parameter given a priori and/or learned.

SGD with L1 and L2 Regularization

- An $L2$ regularization is implemented in stochastic gradient descent by updating each weight w_i after a batch by:

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- An $L1$ regularizer can be implemented by updating each weight after a batch by:

$$w_i := \text{sign}(w_i) * \max(0, |w_i| - \eta * \lambda * m/|Es|).$$

This is called **iterative soft-thresholding**

Cross Validation

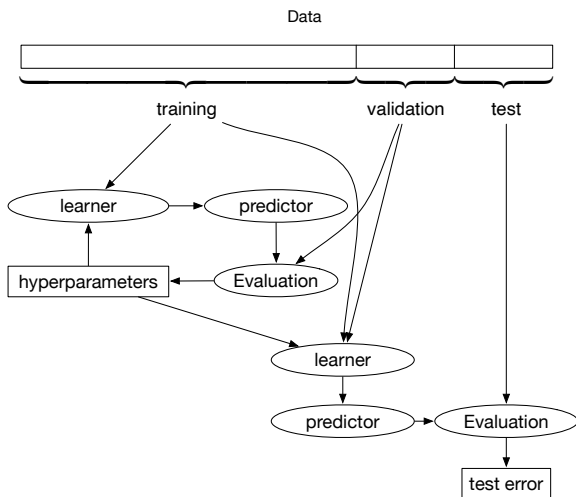
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- A **hyperparameter** is a parameter used to define what is being optimized, or how it is optimized.
- Use the new training set to train on. Select the hyperparameters that work best on the validation set.

Cross Validation

Cross validation: use some of the non test set as a surrogate for test data:

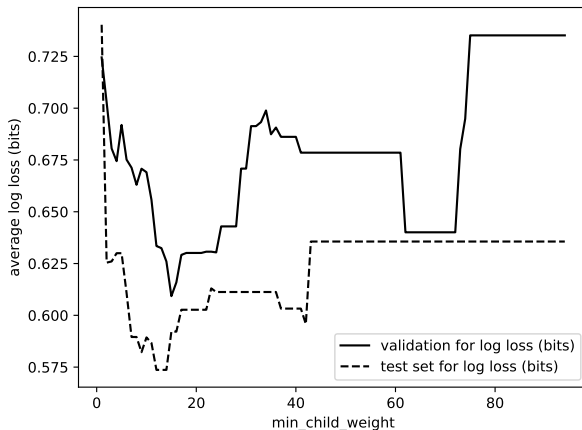


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minimum number of examples that needs to be in a child for decision-tree learning.

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 - ▶ For i from 1 to k :
 - train on $E_s \setminus E_i$ evaluate on E_i
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- If $k = 10$, during hyperparameter tuning, 90% of the training examples are used for training and 10% of the examples for validation.
It does this 10 times, so each example is used once in a validation set.