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- Fitting the training set better does not mean fitting the test set or better predictions of future cases

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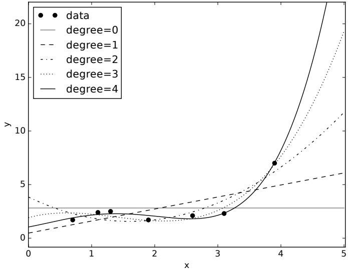
- Ratings are a mix of quality and luck. Lots of data averages out luck.

Example of Overfitting: Model complexity

Polynomials of various degrees can be used to fit data:



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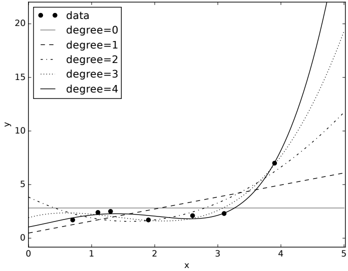

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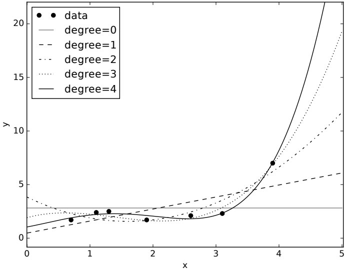

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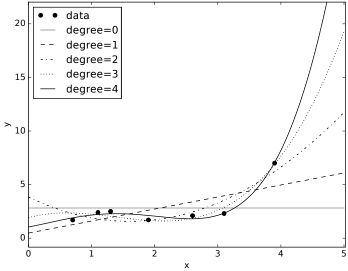

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- Noise: inherent error due to the data depending on features not modeled or the process generating the data is inherently stochastic.

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  - Don't need to store pseudo-examples, just the sufficient statistics: pseudocounts.

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A theoretical justification of pseudocounts is given in Chapter 10.

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- Find a predictor $\widehat{Y}$ to minimize

$$\left(\sum_e loss(\widehat{Y}(e), Y(e))\right) + \lambda * regularizer(\widehat{Y})$$

  - $loss(\widehat{Y}(e), Y(e))$ is the loss of example e for predictor $\widehat{Y}$
  - $regularizer(\widehat{Y})$ is a penalty term that penalizes complexity.
  - The regularization parameter, λ, trades off fit-to-data and model simplicity

Regularization

- In decision tree learning, one complexity measure is the number of leaves in a decision tree.
- When building a decision tree, you could optimize the sum of a loss plus a function of the size of the decision tree, minimizing

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- When greedily splitting, a single split is worthwhile if it reduces the sum of losses by γ.

L1 and L2 Regularization

Linear/logistic regression, minimize sum-of-squares:

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λ is a parameter given a priori and/or learned.

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- An $L2$ regularization is implemented in stochastic gradient descent by updating each weight w_i after a batch by:

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- An $L1$ regularizer can be implemented by updating each weight after a batch by:

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  This is called iterative soft-thresholding

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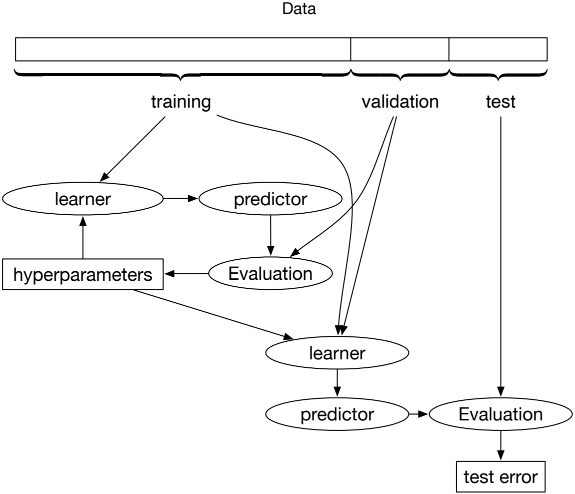
- A hyperparameter is a parameter used to define what is being optimized, or how it is optimized.
- Use the new training set to train on. Select the hyperparameters that work best on the validation set.

Cross Validation

Cross validation: use some of the non test set as a surrogate for test data:



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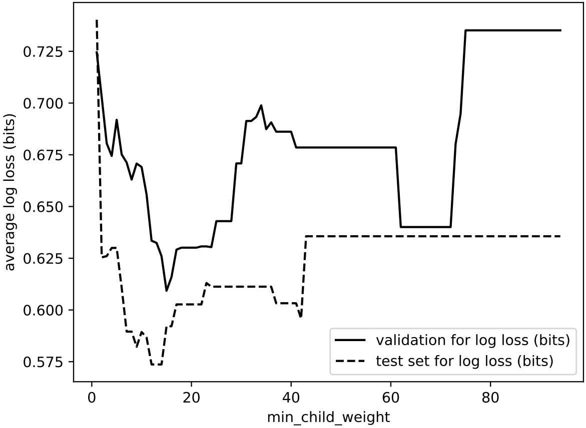

minimum number of examples that needs to be in a child for decision-tree learning.

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- If $k = 10$, during hyperparameter tuning, 90% of the training examples are used for training and 10% of the examples for validation.
  It does this 10 times, so each example is used once in a validation set.