Learning Objectives

At the end of the class you should be able to:

- identify a supervised learning problem
- characterize how the prediction is a function of the error measure
- avoid mixing the training and test sets

Supervised Learning

Given:

- a set of inputs features $X_1, \ldots, X_n$
- a set of target features $Y_1, \ldots, Y_k$
- a set of training examples where the values for the input features and the target features are given for each example
- a new example, where only the values for the input features are given

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- classification when the Y_i are discrete
- regression when the Y_i are continuous

Example Data Representations

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Two representations of the same data:

— Y is the length of trip chosen.

— Each Y_i is an indicator variable that has value 1 if the chosen length is i, and is 0 otherwise.

Example	Y
e_1	1
e_2	6
e_3	6
e_4	2
e_5	1

Example	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
e_1	1	0	0	0	0	0
e_2	0	0	0	0	0	1
e_3	0	0	0	0	0	1
e_4	0	1	0	0	0	0
e_5	1	0	0	0	0	0

What is a prediction?

Evaluating Predictions

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- p_e is the predicted value of target feature on example e.
- The error of the prediction is a measure of how close p_e is to o_e.
- There are many possible errors that could be measured.

Sometimes p_e can be a real number even though o_e can only have a few values.

Measures of error

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- number wrong: $L_0(E) = \#\{e : o_e \neq p_e\}$
- A cost-based error takes into account costs of errors.

Measures of error (cont.)

With binary feature: $o_e \in \{0, 1\}$:

- likelihood of the data

$$\prod_{e \in E} p_e^{o_e} (1 - p_e)^{(1 - o_e)}$$

Measures of error (cont.)

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in terms of information:

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in terms of information:
It is negative of number of bits to encode the data given a code based on p_e.

Information theory overview

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- Can we do better?

Information and Probability

Consider a code to distinguish elements of $\{a, b, c, d\}$ with

$$P(a) = \frac{1}{2}, P(b) = \frac{1}{4}, P(c) = \frac{1}{8}, P(d) = \frac{1}{8}$$

Consider the code:

a	0	b	10	c	110	d	111

This code uses bits.

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The string *aacabbda* has code

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The string *aacabbda* has code 00110010101110.
The code 0111110010100 represents string *adcabba*

Information Content

- To identify x, we need $-\log_2 P(x)$ bits.
- Give a distribution over a set, to a identify a member, the expected number of bits

  $$\sum_x -P(x) \times \log_2 P(x).$$

  is the information content or entropy of the distribution.
- The expected number of bits it takes to describe a distribution given evidence e:

  $$I(e) = \sum_x -P(x|e) \times \log_2 P(x|e).$$

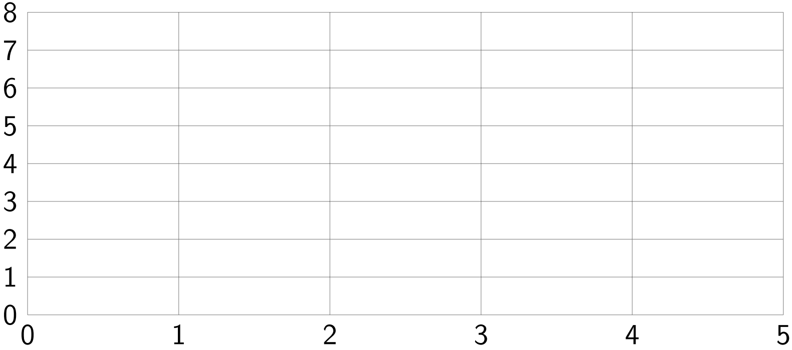
Information Gain

Given a test that can distinguish the cases where α is true from the cases where α is false, the information gain from this test is:

$$I(true) - (P(\alpha) \times I(\alpha) + P(\neg\alpha) \times I(\neg\alpha)).$$

- $I(true)$ is the expected number of bits needed before the test
- $P(\alpha) \times I(\alpha) + P(\neg\alpha) \times I(\neg\alpha)$ is the expected number of bits after the test.

Linear Predictions



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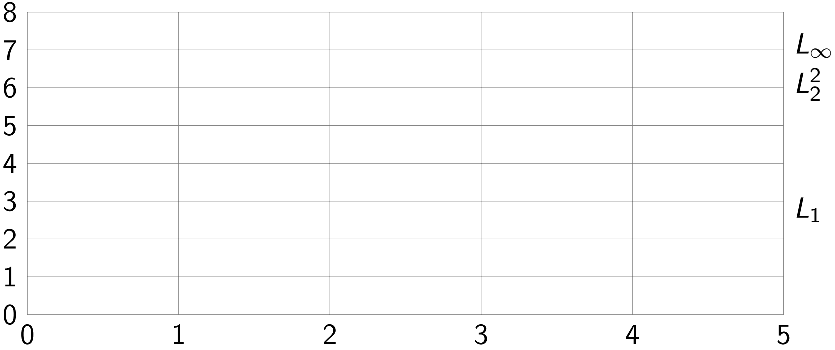

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But that doesn't mean that these predictions minimize the error for future predictions....

Training and Test Sets

To evaluate how well a learner will work on future predictions, we divide the examples into:

- training examples that are used to train the learner
- test examples that are used to evaluate the learner

...these must be kept separate.