Integrity Constraints

- In the electrical domain, what if we predict that a light should be on, but observe that it isn't?
  What can we conclude?
- We will expand the definite clause language to include integrity constraints which are rules that imply *false*, where *false* is an atom that is false in all interpretations.
- This allows proof by contradiction.
- A definite clause knowledge base is always consistent. This won't be true with the rules that imply *false*.

Horn clauses

- An integrity constraint is a clause of the form

  $$false \leftarrow a_1 \wedge \ldots \wedge a_k$$

  where the a_i are atoms and *false* is a special atom that is false in all interpretations.
- A Horn clause is either a definite clause or an integrity constraint.

Negative Conclusions

- Negations can follow from a Horn clause KB.
- The negation of α, written $\neg\alpha$ is a formula that
  - is true in interpretation I if α is false in I, and
  - is false in interpretation I if α is true in I.
- Example:

$$KB = \left\{ \begin{array}{l} false \leftarrow a \wedge b. \\ a \leftarrow c. \\ b \leftarrow c. \end{array} \right\} \qquad KB \models \neg c.$$

Disjunctive Conclusions

- Disjunctions can follow from a Horn clause KB.
- The disjunction of α and β, written $\alpha \vee \beta$, is
  - true in interpretation I if α is true in I or β is true in I (or both are true in I).
  - false in interpretation I if α and β are both false in I.
- Example:

$$KB = \left\{ \begin{array}{l} false \leftarrow a \wedge b. \\ a \leftarrow c. \\ b \leftarrow d. \end{array} \right\} \qquad KB \models \neg c \vee \neg d.$$

Questions and Answers in Horn KBs

- An assumable is an atom whose negation you are prepared to accept as part of a (disjunctive) answer.
- A conflict of KB is a set of assumables that, given KB imply *false*.
- A minimal conflict is a conflict such that no strict subset is also a conflict.

Conflict Example

Example: If $\{c, d, e, f, g, h\}$ are the assumables

$$KB = \left\{ \begin{array}{l} false \leftarrow a \wedge b. \\ a \leftarrow c. \\ b \leftarrow d. \\ b \leftarrow e. \end{array} \right\}$$

What are some conflicts?

- $\{c, d\}$ is a conflict
- $\{c, e\}$ is a conflict
- $\{c, d, e, h\}$ is a conflict

What are the minimal conflicts?

Using Conflicts for Diagnosis

- Assume that the user is able to observe whether a light is lit or dark and whether a power outlet is dead or live.
- A light can't be both lit and dark. An outlet can't be both live and dead:

$$false \leftarrow dark\_l_1 \;\&\; lit\_l_1.$$

$$false \leftarrow dark\_l_2 \;\&\; lit\_l_2.$$

$$false \leftarrow dead\_p_1 \;\&\; live\_p_2.$$

- Assume the individual components are working correctly:

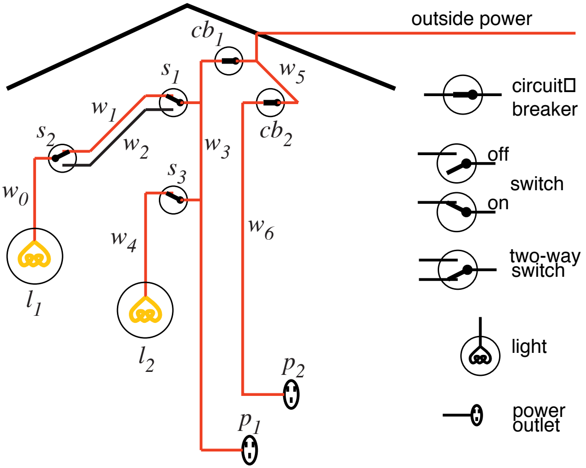
$$assumable\ ok\_l_1.$$

$$assumable\ ok\_s_2.$$

$$\ldots$$

- Suppose switches s_1, s_2, and s_3 are all up:
  $up\_s_1.\ up\_s_2.\ up\_s_3.$

Electrical Environment



in aipython.org, run code at the end of

`logicAssumables.py`

Representing the Electrical Environment

$light\_l_1.$

$light\_l_2.$

$up\_s_1.$

$up\_s_2.$

$up\_s_3.$

$live\_outside.$

$lit\_l_1 \leftarrow live\_w_0 \wedge ok\_l_1.$

$live\_w_0 \leftarrow live\_w_1 \wedge up\_s_2 \wedge ok\_s_2.$

$live\_w_0 \leftarrow live\_w_2 \wedge down\_s_2 \wedge ok\_s_2.$

$live\_w_1 \leftarrow live\_w_3 \wedge up\_s_1 \wedge ok\_s_1.$

$live\_w_2 \leftarrow live\_w_3 \wedge down\_s_1 \wedge ok\_s_1.$

$lit\_l_2 \leftarrow live\_w_4 \wedge ok\_l_2.$

$live\_w_4 \leftarrow live\_w_3 \wedge up\_s_3 \wedge ok\_s_3.$

$live\_p_1 \leftarrow live\_w_3.$

$live\_w_3 \leftarrow live\_w_5 \wedge ok\_cb_1.$

$live\_p_2 \leftarrow live\_w_6.$

$live\_w_6 \leftarrow live\_w_5 \wedge ok\_cb_2.$

$live\_w_5 \leftarrow live\_outside.$

- If the user has observed l_1 and l_2 are both dark:

$$dark\_l_1.\ dark\_l_2.$$

- There are two minimal conflicts:

  $\{ok\_cb_1, ok\_s_1, ok\_s_2, ok\_l_1\}$ and

  $\{ok\_cb_1, ok\_s_3, ok\_l_2\}$.

- You can derive:

  $\neg ok\_cb_1 \vee \neg ok\_s_1 \vee \neg ok\_s_2 \vee \neg ok\_l_1$

  $\neg ok\_cb_1 \vee \neg ok\_s_3 \vee \neg ok\_l_2$.

- Either cb_1 is broken or there is one of six double faults.

Diagnoses

- A consistency-based diagnosis is a set of assumables that has at least one element in each conflict.
- A minimal diagnosis is a diagnosis such that no subset is also a diagnosis.
- Intuitively, one of the minimal diagnoses must hold. A diagnosis holds if all of its elements are false.
- Example: For the proceeding example there are seven minimal diagnoses: $\{ok\_cb_1\}$, $\{ok\_s_1, ok\_s_3\}$, $\{ok\_s_1, ok\_l_2\}$, $\{ok\_s_2, ok\_s_3\}$,...

Recall: top-down consequence finding

To solve the query $?q_1 \wedge \ldots \wedge q_k$:

> $ac :=$ "$yes \leftarrow q_1 \wedge \ldots \wedge q_k$"
> **repeat**
> > **select** atom a_i from the body of ac;
> > **choose** clause C from KB with a_i as head;
> > replace a_i in the body of ac by the body of C
>
> **until** ac is an answer.

Implementing conflict finding: top down

- Query is *false*.
- Don't select an atom that is assumable.
- Stop when all of the atoms in the body of the generalised query are assumable:
  - this is a conflict

Example

$$false \leftarrow a.$$
$$a \leftarrow b \,\&\, c.$$
$$b \leftarrow d.$$
$$b \leftarrow e.$$
$$c \leftarrow f.$$
$$c \leftarrow g.$$
$$e \leftarrow h \,\&\, w.$$
$$e \leftarrow g.$$
$$w \leftarrow f.$$

assumable d, f, g, h.

Bottom-up Conflict Finding

- Conclusions are pairs $\langle a, A \rangle$, where a is an atom and A is a set of assumables that imply a.
- Initially, conclusion set $C = \{\langle a, \{a\} \rangle : a \text{ is assumable}\}$.
- If there is a rule $h \leftarrow b_1 \wedge \ldots \wedge b_m$ such that for each b_i there is some A_i such that $\langle b_i, A_i \rangle \in C$, then $\langle h, A_1 \cup \ldots \cup A_m \rangle$ can be added to C.
- If $\langle a, A_1 \rangle$ and $\langle a, A_2 \rangle$ are in C, where $A_1 \subset A_2$, then $\langle a, A_2 \rangle$ can be removed from C.
- If $\langle \mathit{false}, A_1 \rangle$ and $\langle a, A_2 \rangle$ are in C, where $A_1 \subseteq A_2$, then $\langle a, A_2 \rangle$ can be removed from C.

Bottom-up Conflict Finding Code

$C := \{\langle a, \{a\}\rangle : a \text{ is assumable }\};$
repeat
    select clause "$h \leftarrow b_1 \wedge \ldots \wedge b_m$" in T such that
        $\langle b_i, A_i\rangle \in C$ for all i and
        there is no $\langle h, A'\rangle \in C$ or $\langle \mathit{false}, A'\rangle \in C$
            such that $A' \subseteq A$ where $A = A_1 \cup \ldots \cup A_m$
    $C := C \cup \{\langle h, A\rangle\}$
    Remove any elements of C that can now be pruned
until no more selections are possible

Assumption-based Reasoning

Often we want our agents to make assumptions rather than doing deduction from their knowledge. For example:

- In abduction an agent makes assumptions to explain observations. For example, it hypothesizes what could be wrong with a system to produce the observed symptoms.
- In default reasoning an agent makes assumptions of normality to make predictions. For example, the delivery robot may want to assume Mary is in her office, even if it isn't always true.

Design and Recognition

Two different tasks use assumption-based reasoning:

- Design The aim is to design an artifact or plan. The designer can select whichever design they like that satisfies the design criteria.
- Recognition The aim is to find out what is true based on observations. If there are a number of possibilities, the recognizer can't select the one they like best. The underlying reality is fixed; the aim is to find out what it is.

Compare: Recognizing a disease with designing a treatment. Designing a meeting time with determining when it is.

An Assumption-based Framework

The assumption-based framework is defined in terms of two sets of formulae:

- F is a set of closed formula called the facts.
  These are formulae that are given as true in the world.
  Assume F are Horn clauses.
- H is a set of formulae called the possible hypotheses or assumables. Ground instance of the possible hypotheses can be assumed if consistent.

Making Assumptions

- A scenario of $\langle F, H\rangle$ is a set D of ground instances of elements of H such that $F \cup D$ is satisfiable.
- An explanation of g from $\langle F, H\rangle$ is a scenario that, together with F, implies g.
  D is an explanation of g if $F \cup D \models g$ and $F \cup D \not\models \mathit{false}$.
  A minimal explanation is an explanation such that no strict subset is also an explanation.
- An extension of $\langle F, H\rangle$ is the set of logical consequences of F and a maximal scenario of $\langle F, H\rangle$.

Example

$a \leftarrow b \wedge c.$
$b \leftarrow e.$
$b \leftarrow h.$
$c \leftarrow g.$
$c \leftarrow f.$
$d \leftarrow g.$
$\mathit{false} \leftarrow e \wedge d.$
$f \leftarrow h \wedge m.$
assumable e, h, g, m, n.

- Is $\{e, m, n\}$ a scenario? yes
- Is $\{e, g, m\}$ a scenario. no
- Is $\{h, m\}$ an explanation for a. yes
- Is $\{e, h, m\}$ an explanation for a. yes
- Is $\{e, g, h, m\}$ an explanation for a. no
- Is $\{e, h, m, n\}$ a maximal scenario. yes
- Is $\{h, g, m, n\}$ a maximal scenario. yes

Default Reasoning and Abduction

There are two strategies for using the assumption-based framework:

- Default reasoning Where the truth of g is unknown and is to be determined.
  An explanation for g corresponds to an argument for g.
- Abduction Where g is given, and we are interested in explaining it. g could be an observation in a recognition task or a design goal in a design task.

Give observations, we typically do abduction, then default reasoning to find consequences.

Computing Explanations

To find assumables to imply the query $?q_1 \wedge \ldots \wedge q_k$:

> $ac :=$ "$yes \leftarrow q_1 \wedge \ldots \wedge q_k$"
> **repeat**
>> **select** non-assumable atom a_i from the body of ac
>> **choose** clause C from KB with a_i as head
>> replace a_i in the body of ac by the body of C
>
> **until** all atoms in the body of ac are assumable.

To find an explanation of query $?q_1 \wedge \ldots \wedge q_k$:

- find assumables to imply $?q_1 \wedge \ldots \wedge q_k$
- ensure that no subset of the assumables found implies *false*