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- This allows proof by contradiction.
- A definite clause knowledge base is always consistent. This won't be true with the rules that imply false.


## Horn clauses

- An integrity constraint is a clause of the form

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\text { false } \leftarrow a_{1} \wedge \ldots \wedge a_{k}
$$

where the $a_{i}$ are atoms and false is a special atom that is false in all interpretations.

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- A Horn clause is either a definite clause or an integrity constraint.


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- A minimal conflict is a conflict such that no strict subset is also a conflict.


## Conflict Example

Example: If $\{c, d, e, f, g, h\}$ are the assumables

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\end{aligned}
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- Assume the individual components are working correctly: assumable ok_l. assumable ok_s 2 $_{2}$.
- Suppose switches $s_{1}, s_{2}$, and $s_{3}$ are all up: up_s1. up_s2. up_s3.


## Electrical Environment


in aipython.org, run code at the end of
logicAssumables.py

Representing the Electrical Environment

|  | $l i t \_l_{1} \leftarrow l_{\text {live_ }} w_{0} \wedge$ ok_ $l_{1}$. |
| :---: | :---: |
|  | live_ $w_{0} \leftarrow$ live_ $w_{1} \wedge u p_{-} s_{2} \wedge o k_{-} s_{2}$. <br> live_ $w_{0} \leftarrow$ live_ $_{2} \wedge$ down_s $s_{2} \wedge o k_{-} s_{2}$. |
| light $\iota_{1}$. | live $w_{1} \leftarrow \mathrm{live}_{-} w_{3} \wedge u p_{-} s_{1} \wedge o k_{-s_{1}}$. |
| light_2. | live_ $w_{2} \leftarrow$ live_ $^{\prime} w_{3} \wedge$ down_s $s_{1} \wedge$ ok_s $s_{1}$. |
| $u p_{-} s_{1}$. | $l_{\text {lit }}^{1} l_{2} \leftarrow \mathrm{live}_{-} w_{4} \wedge$ ok_l $l_{2}$. |
| $u p_{-} s_{2}$. | live_w $w_{4} \leftarrow$ live_w $w_{3} \wedge u p_{-} s_{3} \wedge$ ok_s $s_{3}$. |
| up_S3. | live_ $p_{1} \leftarrow$ live_w ${ }_{3}$. |
| live_outside. | $l_{\text {live_ }} w_{3} \leftarrow$ live_ $w_{5} \wedge$ ok_cb ${ }_{1}$. |
|  | live_ $p_{2} \leftarrow$ live_w6. |
|  | live_ $w_{6} \leftarrow$ live_ $w_{5} \wedge$ ok_cb ${ }_{2}$. |
|  | live_ $w_{5} \leftarrow$ live_outside. |

- If the user has observed $I_{1}$ and $I_{2}$ are both dark:

$$
\operatorname{dark}_{-} l_{1} . \text { dark_}_{2} .
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$$
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& \neg o k_{-} c b_{1} \vee \neg o k_{-} s_{1} \vee \neg o k_{-} s_{2} \vee \neg o k_{-} I_{1} \\
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- Either
- If the user has observed $I_{1}$ and $I_{2}$ are both dark:

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- Either $c b_{1}$ is broken or there is one of six double faults.


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- Example: For the proceeding example there are seven minimal diagnoses: $\left\{o k_{-} c b_{1}\right\},\left\{o k_{-} s_{1}, o k_{-} s_{3}\right\},\left\{o k_{-} s_{1}, o k_{-} l_{2}\right\}$, $\left\{o k_{-} s_{2}, o k_{-} s_{3}\right\}, \ldots$


## Recall: top-down consequence finding

To solve the query $? q_{1} \wedge \ldots \wedge q_{k}$ :

$$
a c:=" y e s \leftarrow q_{1} \wedge \ldots \wedge q_{k} "
$$

repeat
select atom $a_{i}$ from the body of $a c$; choose clause $C$ from $K B$ with $a_{i}$ as head; replace $a_{i}$ in the body of $a c$ by the body of $C$ until $a c$ is an answer.

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## Implementing conflict finding: top down

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- this is a conflict


## Example

$$
\begin{aligned}
& \text { false } \leftarrow a . \\
& a \leftarrow b \& c . \\
& b \leftarrow d . \\
& b \leftarrow e \\
& c \leftarrow f \\
& c \leftarrow g . \\
& e \leftarrow h \& w . \\
& e \leftarrow g . \\
& w \leftarrow f . \\
& \text { assumable } d, f, g, h .
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## Bottom-up Conflict Finding

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## Bottom-up Conflict Finding Code

$C:=\{\langle a,\{a\}\rangle: a$ is assumable $\} ;$

## repeat

select clause " $h \leftarrow b_{1} \wedge \ldots \wedge b_{m}$ " in $T$ such that $\left\langle b_{i}, A_{i}\right\rangle \in C$ for all $i$ and there is no $\left\langle h, A^{\prime}\right\rangle \in C$ or $\left\langle\right.$ false, $\left.A^{\prime}\right\rangle \in C$ such that $A^{\prime} \subseteq A$ where $A=A_{1} \cup \ldots \cup A_{m}$
$C:=C \cup\{\langle h, A\rangle\}$
Remove any elements of $C$ that can now be pruned until no more selections are possible

## Assumption-based Reasoning

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- In abduction an agent makes assumptions to explain observations. For example, it hypothesizes what could be wrong with a system to produce the observed symptoms.
- In default reasoning an agent makes assumptions of normality to make predictions. For example, the delivery robot may want to assume Mary is in her office, even if it isn't always true.


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Compare: Recognizing a disease with designing a treatment. Designing a meeting time with determining when it is.


## An Assumption-based Framework

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These are formulae that are given as true in the world. Assume $F$ are Horn clauses.

- $H$ is a set of formulae called the possible hypotheses or assumables. Ground instance of the possible hypotheses can be assumed if consistent.


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## Making Assumptions

- A scenario of $\langle F, H\rangle$ is a set $D$ of ground instances of elements of $H$ such that $F \cup D$ is satisfiable.
- An explanation of $g$ from $\langle F, H\rangle$ is a scenario that, together with $F$, implies $g$.
$D$ is an explanation of $g$ if $F \cup D \models g$ and $F \cup D \not \models f$ false. A minimal explanation is an explanation such that no strict subset is also an explanation.
- An extension of $\langle F, H\rangle$ is the set of logical consequences of $F$ and a maximal scenario of $\langle F, H\rangle$.


## Example

```
\(a \leftarrow b \wedge c . \quad \bullet\) Is \(\{e, m, n\}\) a scenario?
\(b \leftarrow e\).
\(b \leftarrow h\).
\(c \leftarrow g\).
\(c \leftarrow f\).
\(d \leftarrow g\).
false \(\leftarrow e \wedge d\).
\(f \leftarrow h \wedge m\).
assumable \(e, h, g, m, n\).
```


## Example

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\(a \leftarrow b \wedge c\).
- Is \(\{e, m, n\}\) a scenario? yes
\(b \leftarrow e\).
\(b \leftarrow h\).
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\(c \leftarrow f\).
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- Is $\{e, m, n\}$ a scenario? yes
- Is $\{e, g, m\}$ a scenario. no
- Is $\{h, m\}$ an explanation for $a$. yes
- Is $\{e, h, m\}$ an explanation for $a$. yes
- Is $\{e, g, h, m\}$ an explanation for $a$.

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\begin{aligned}
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& b \leftarrow h \text {. } \\
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& \text { false } \leftarrow e \wedge d \text {. } \\
& f \leftarrow h \wedge m \text {. } \\
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## Default Reasoning and Abduction

There are two strategies for using the assumption-based framework:

- Default reasoning Where the truth of $g$ is unknown and is to be determined.
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Give observations, we typically do abduction, then default reasoning to find consequences.


## Computing Explanations

To find assumables to imply the query $? q_{1} \wedge \ldots \wedge q_{k}$ :

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a c:=" y e s \leftarrow q_{1} \wedge \ldots \wedge q_{k} "
$$

repeat
select non-assumable atom $a_{i}$ from the body of ac

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To find an explanation of query $? q_{1} \wedge \ldots \wedge q_{k}$ :

- find assumables to imply ? $q_{1} \wedge \ldots \wedge q_{k}$
- ensure that no subset of the assumables found implies false

