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- We will expand the definite clause language to include **integrity constraints** which are rules that imply *false*, where *false* is an atom that is false in all interpretations.
- This allows proof by contradiction.
- A definite clause knowledge base is always consistent. This won't be true with the rules that imply *false*.

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$$false \leftarrow a_1 \wedge \dots \wedge a_k$$

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- A **Horn clause** is either a definite clause or an integrity constraint.

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- A **minimal conflict** is a conflict such that no strict subset is also a conflict.

# Conflict Example

**Example:** If  $\{c, d, e, f, g, h\}$  are the assumables

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*false*  $\leftarrow$  *dark*<sub>l1</sub> & *lit*<sub>l1</sub>.

*false*  $\leftarrow$  *dark*<sub>l2</sub> & *lit*<sub>l2</sub>.

*false*  $\leftarrow$  *dead*<sub>p1</sub> & *live*<sub>p2</sub>.

- Assume the individual components are working correctly:

*assumable ok*<sub>l1</sub>.

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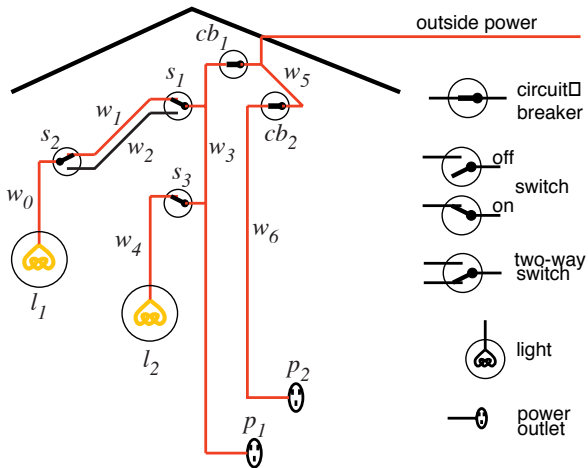
$assumable \ ok_{s_2}.$

...

- Suppose switches  $s_1$ ,  $s_2$ , and  $s_3$  are all up:

$up_{s_1}. \ up_{s_2}. \ up_{s_3}.$

# Electrical Environment



in [aipython.org](http://aipython.org), run code at the end of  
`logicAssumables.py`

# Representing the Electrical Environment

$lit\_l_1 \leftarrow live\_w_0 \wedge ok\_l_1.$   
 $live\_w_0 \leftarrow live\_w_1 \wedge up\_s_2 \wedge ok\_s_2.$   
 $live\_w_0 \leftarrow live\_w_2 \wedge down\_s_2 \wedge ok\_s_2.$   
 $light\_l_1.$   
 $live\_w_1 \leftarrow live\_w_3 \wedge up\_s_1 \wedge ok\_s_1.$   
 $light\_l_2.$   
 $live\_w_2 \leftarrow live\_w_3 \wedge down\_s_1 \wedge ok\_s_1.$   
 $up\_s_1.$   
 $lit\_l_2 \leftarrow live\_w_4 \wedge ok\_l_2.$   
 $up\_s_2.$   
 $live\_w_4 \leftarrow live\_w_3 \wedge up\_s_3 \wedge ok\_s_3.$   
 $up\_s_3.$   
 $live\_p_1 \leftarrow live\_w_3.$   
 $live\_outside.$   
 $live\_w_3 \leftarrow live\_w_5 \wedge ok\_cb_1.$   
 $live\_p_2 \leftarrow live\_w_6.$   
 $live\_w_6 \leftarrow live\_w_5 \wedge ok\_cb_2.$   
 $live\_w_5 \leftarrow live\_outside.$

- If the user has observed  $l_1$  and  $l_2$  are both dark:

*dark\_1. dark\_2.*

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- You can derive:



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- Either  $cb_1$  is broken or there is one of six double faults.

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- **Example:** For the proceeding example there are seven minimal diagnoses:  $\{ok\_cb_1\}$ ,  $\{ok\_s_1, ok\_s_3\}$ ,  $\{ok\_s_1, ok\_l_2\}$ ,  $\{ok\_s_2, ok\_s_3\}, \dots$

## Recall: top-down consequence finding

To solve the query  $?q_1 \wedge \dots \wedge q_k$ :

$ac := \text{“yes} \leftarrow q_1 \wedge \dots \wedge q_k\text{”}$

**repeat**

**select** atom  $a_i$  from the body of  $ac$ ;

**choose** clause  $C$  from  $KB$  with  $a_i$  as head;

    replace  $a_i$  in the body of  $ac$  by the body of  $C$

**until**  $ac$  is an answer.

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# Example

$false \leftarrow a.$

$a \leftarrow b \& c.$

$b \leftarrow d.$

$b \leftarrow e.$

$c \leftarrow f.$

$c \leftarrow g.$

$e \leftarrow h \& w.$

$e \leftarrow g.$

$w \leftarrow f.$

assumable  $d, f, g, h.$

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# Bottom-up Conflict Finding Code

```
C := {⟨a, {a}⟩ : a is assumable };  
repeat  
  select clause “ $h \leftarrow b_1 \wedge \dots \wedge b_m$ ” in  $T$  such that  
    ⟨ $b_i, A_i$ ⟩ ∈ C for all  $i$  and  
    there is no ⟨ $h, A'$ ⟩ ∈ C or ⟨false,  $A'$ ⟩ ∈ C  
      such that  $A' \subseteq A$  where  $A = A_1 \cup \dots \cup A_m$   
  C := C ∪ {⟨ $h, A$ ⟩}  
  Remove any elements of C that can now be pruned  
until no more selections are possible
```



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- In **abduction** an agent makes assumptions to explain observations. For example, it hypothesizes what could be wrong with a system to produce the observed symptoms.
- In **default reasoning** an agent makes assumptions of normality to make predictions. For example, the delivery robot may want to assume Mary is in her office, even if it isn't always true.

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Designing a meeting time with determining when it is.

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Assume  $F$  are Horn clauses.
- $H$  is a set of formulae called the **possible hypotheses** or **assumables**. Ground instance of the possible hypotheses can be assumed if consistent.



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# Making Assumptions

- A **scenario** of  $\langle F, H \rangle$  is a set  $D$  of ground instances of elements of  $H$  such that  $F \cup D$  is satisfiable.
- An **explanation** of  $g$  from  $\langle F, H \rangle$  is a scenario that, together with  $F$ , implies  $g$ .  
 $D$  is an explanation of  $g$  if  $F \cup D \models g$  and  $F \cup D \not\models \text{false}$ .  
A **minimal explanation** is an explanation such that no strict subset is also an explanation.
- An **extension** of  $\langle F, H \rangle$  is the set of logical consequences of  $F$  and a maximal scenario of  $\langle F, H \rangle$ .

# Example

$a \leftarrow b \wedge c.$

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$b \leftarrow h.$

$c \leftarrow g.$

$c \leftarrow f.$

$d \leftarrow g.$

$\text{false} \leftarrow e \wedge d.$

$f \leftarrow h \wedge m.$

assumable  $e, h, g, m, n.$

- Is  $\{e, m, n\}$  a scenario?

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# Default Reasoning and Abduction

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- **Abduction** Where  $g$  is given, and we are interested in explaining it.  $g$  could be an observation in a recognition task or a design goal in a design task.

Give observations, we typically do abduction, then default reasoning to find consequences.

To find assumables to imply the query  $?q_1 \wedge \dots \wedge q_k$ :

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To find an explanation of query  $?q_1 \wedge \dots \wedge q_k$ :

- find assumables to imply  $?q_1 \wedge \dots \wedge q_k$
- ensure that no subset of the assumables found implies *false*