Logic and Propositions

"Consequently he who wishes to attain to human perfection, must therefore first study Logic, next the various branches of Mathematics in their proper order, then Physics, and lastly Metaphysics."

Maimonides 1135–1204

Propositions

- A proposition is a sentence, written in a language, that has a truth value – it is true or false – in a world. A proposition is built from atomic propositions (atoms) and logical connectives.
- Propositions can be built from simpler propositions using logical connectives.

Propositional Calculus Syntax

- An atomic proposition – atom – is a symbol, written as sequences of letters, digits, and the underscore (_) and start with a lower-case letter.
  E.g., a, ai_is_fun, lit_l_1, $live\_outside$, $mimsy$, $sunny$.
- A proposition or logical formula is either
  - an atomic proposition or
  - a compound proposition of the form

$\neg p$	"not p"	negation of p
$p \wedge q$	"p and q"	conjunction of p and q
$p \vee q$	"p or q"	disjunction of p and q
$p \rightarrow q$	"p implies q"	implication of q from p
$p \leftarrow q$	"p if q"	implication of p from q
$p \leftrightarrow q$	"p if and only if q"	equivalence of p and q
$p \oplus q$	"p XOR q"	exclusive-or of p and q

    where p and q are propositions.
- The operators $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftarrow$, $\leftrightarrow$, and $\oplus$ are logical connectives.

Why propositions?

- Logical formulae are modular statements of what is (known to be) true
- It is easier to check correctness and debug formulae than tables of what could be true
- We can exploit the Boolean nature for efficient reasoning
- We need a language for asking queries (of what follows in all models) that may be more complicated than asking for the value of a variable
- It is easy to incrementally add formulae
- It can be extended to infinitely many propositions with infinite domains (using logical quantification)

Semantics of the Propositional Calculus

- An interpretation – or possible world – is an assignment of true or false to each variable.
- An interpretation is defined by function π that maps atoms to $\{true, false\}$.
  If $\pi(a)$=*true*, atom a is true in the interpretation.
  If $\pi(a)$=*false*, atom a is false in the interpretation.
- Truth of a compound proposition in an interpretation is defined in terms of the truth of its components:

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftarrow q$	$p \leftrightarrow q$	$p \oplus q$
true	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*true*	*false*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*	*false*	*true*
false	*false*	*true*	*false*	*false*	*true*	*true*	*true*	*false*

- Propositions can have different truth values in different interpretations.

Models and Logical Consequence

- A model of a set of clauses is an interpretation in which all the clauses are *true*.
- If KB is a set of propositions, proposition g is a logical consequence of KB, written $KB \models g$, if g is *true* in every model of KB.
- That is, $KB \models g$ if there is no interpretation in which KB is *true* and g is *false*.

Simple Example

$$KB = \begin{cases} apple\_eaten \leftarrow bird\_eats\_apple. \\ light\_on \leftarrow night. \\ night. \end{cases}$$

	$apple\_eaten$	$bird\_eats\_apple$	$light\_on$	$night$	model of KB?
I_1	*true*	*true*	*true*	*true*	yes
I_2	*false*	*false*	*false*	*false*	no
I_3	*true*	*true*	*false*	*false*	no
I_4	*false*	*false*	*true*	*true*	yes
I_5	*true*	*false*	*true*	*true*	yes

Which of $apple\_eaten$, $bird\_eats\_apple$, $light\_on$, $night$ logically follow from KB?

$KB \models light\_on$, $KB \models night$,

$KB \not\models apple\_eaten$, $KB \not\models bird\_eats\_apple$

Human's view of semantics

Step 1 Begin with a task domain.

Step 2 Choose atoms in the computer to denote propositions. These atoms have meaning to the KB designer.

Step 3 Tell the system knowledge about the domain.

Step 4 Ask the system questions.

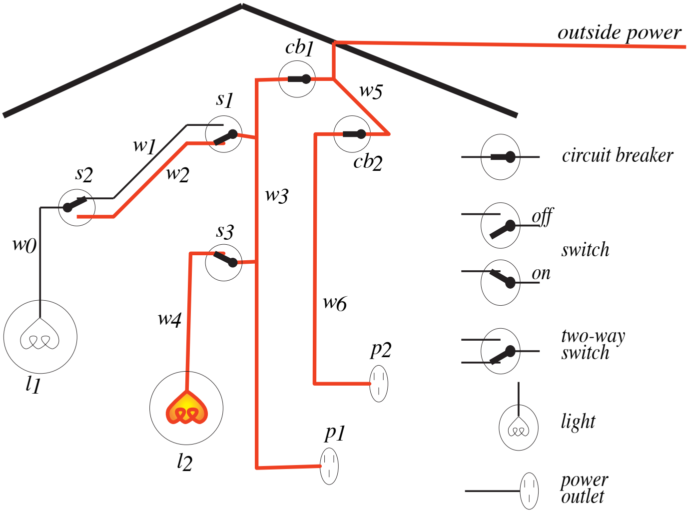
— The system can tell you whether the question is a logical consequence.

— You can interpret the answer with the meaning associated with the atoms.

Computer's view of semantics

- The computer doesn't have access to the intended interpretation.
- All it knows is the knowledge base.
- The computer can determine if a formula is a logical consequence of KB.
- If $KB \models g$ then g must be true in the intended interpretation.
- If $KB \not\models g$ then there is a model of KB in which g is false. This could be the intended interpretation.

Electrical Environment



Role of semantics

In computer:

$$light2\_broken \leftarrow power\_in\_w\_3 \land sw\_3\_up \land unlit\_light2.$$

$$sw\_3\_up.$$

$$power\_in\_w\_3 \leftarrow power\_in\_p\_1.$$

$$unlit\_light2.$$

$$power\_in\_p\_1.$$

In user's mind:

- $light2\_broken$: light #2 is broken
- sw_3_up: switch 3 is up
- $power\_in\_w\_3$: there is power in wire 3
- $unlit\_light2$: light #2 isn't lit
- $power\_in\_p\_1$: outlet p_1 has power

Conclusion: $light2\_broken$

- The computer doesn't know the meaning of the symbols
- The user can interpret symbols using their meaning