Propositions

- A proposition is a sentence, written in a language, that has a truth value – it is true or false – in a world. A proposition is built from atomic propositions (atoms) and logical connectives.
- Propositions can be built from simpler propositions using logical connectives.

Propositional Calculus Syntax

- An atomic proposition – atom – is a symbol, written as sequences of letters, digits, and the underscore (_) and start with a lower-case letter.
  E.g., *a*, *ai_is_fun*, *lit_l*$_1$, *live_outside*, *mimsy*, *sunny*.
- A proposition or logical formula is either
  - an atomic proposition or
  - a compound proposition of the form

$\neg p$	"not p"	negation of p
$p \wedge q$	"p and q"	conjunction of p and q
$p \vee q$	"p or q"	disjunction of p and q
$p \rightarrow q$	"p implies q"	implication of q from p
$p \leftarrow q$	"p if q"	implication of p from q
$p \leftrightarrow q$	"p if and only if q"	equivalence of p and q
$p \oplus q$	"p XOR q"	exclusive-or of p and q

  where p and q are propositions.
- The operators $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftarrow$, $\leftrightarrow$, and $\oplus$ are logical connectives.

Why propositions?

- Logical formulae are modular statements of what is (known to be) true
- It is easier to check correctness and debug formulae than tables of what could be true
- We can exploit the Boolean nature for efficient reasoning
- We need a language for asking queries (of what follows in all models) that may be more complicated than asking for the value of a variable
- It is easy to incrementally add formulae
- It can be extended to infinitely many propositions with infinite domains (using logical quantification)

Semantics of the Propositional Calculus

- An interpretation – or possible world – is an assignment of true or false to each variable.
- An interpretation is defined by function π that maps atoms to $\{true, false\}$.
  If $\pi(a)$=*true*, atom a is true in the interpretation.
  If $\pi(a)$=*false*, atom a is false in the interpretation.
- Truth of a compound proposition in an interpretation is defined in terms of the truth of its components:

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftarrow q$	$p \leftrightarrow q$	$p \oplus q$
true	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*true*	*false*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*	*false*	*true*
false	*false*	*true*	*false*	*false*	*true*	*true*	*true*	*false*

- Propositions can have differenet truth values in different interpretations.

Models and Logical Consequence

- A model of a set of clauses is an interpretation in which all the clauses are *true*.
- If KB is a set of propositions, proposition g is a logical consequence of KB, written $KB \models g$, if g is *true* in every model of KB.
- That is, $KB \models g$ if there is no interpretation in which KB is *true* and g is *false*.

Simple Example

$$KB = \begin{cases} apple\_eaten \leftarrow bird\_eats\_apple. \\ light\_on \leftarrow night. \\ night. \end{cases}$$

	apple_eaten	*bird_eats_apple*	*light_on*	*night*	model of *KB*?
I_1	*true*	*true*	*true*	*true*	yes
I_2	*false*	*false*	*false*	*false*	no
I_3	*true*	*true*	*false*	*false*	no
I_4	*false*	*false*	*true*	*true*	yes
I_5	*true*	*false*	*true*	*true*	yes

Which of *apple_eaten*, *bird_eats_apple*, *light_on*, *night* logically follow from KB?

$KB \models light\_on$, $KB \models night$,

$KB \not\models apple\_eaten$, $KB \not\models bird\_eats\_apple$

Human's view of semantics

Step 1 Begin with a task domain.

Step 2 Choose atoms in the computer to denote propositions. These atoms have meaning to the KB designer.

Step 3 Tell the system knowledge about the domain.

Step 4 Ask the system questions.

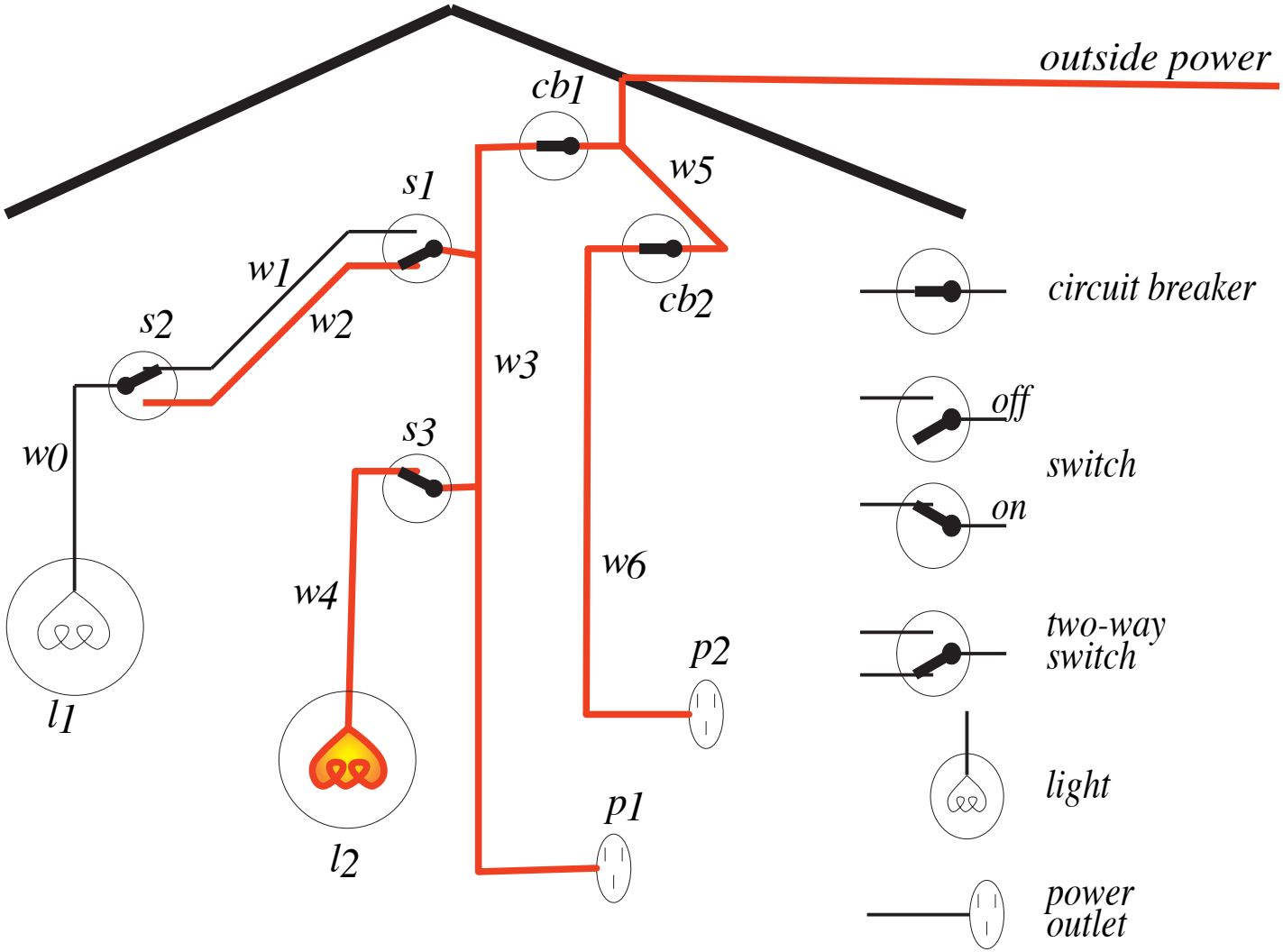
— The system can tell you whether the question is a logical consequence.

— You can interpret the answer with the meaning associated with the atoms.

Computer's view of semantics

- The computer doesn't have access to the intended interpretation.
- All it knows is the knowledge base.
- The computer can determine if a formula is a logical consequence of KB.
- If $KB \models g$ then g must be true in the intended interpretation.
- If $KB \not\models g$ then there is a model of KB in which g is false. This could be the intended interpretation.

Electrical Environment



Role of semantics

In computer:

$light2\_broken \leftarrow power\_in\_w\_3 \wedge sw\_3\_up \wedge unlit\_light2.$

$sw\_3\_up.$

$power\_in\_w\_3 \leftarrow power\_in\_p\_1.$

$unlit\_light2.$

$power\_in\_p\_1.$

In user's mind:

- *light2_broken*: light #2 is broken
- *sw_3_up*: switch 3 is up
- *power_in_w_3*: there is power in wire 3
- *unlit_light2*: light #2 isn't lit
- *power_in_p_1*: outlet *p_1* has power

Conclusion: *light2_broken*

- The computer doesn't know the meaning of the symbols
- The user can interpret symbols using their meaning