Logic and Propositions

"Consequently he who wishes to attain to human perfection, must therefore first study Logic, next the various branches of Mathematics in their proper order, then Physics, and lastly Metaphysics."

Maimonides 1135–1204

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- Propositions can be built from simpler propositions using logical connectives.

Propositional Calculus Syntax

- An atomic proposition – atom – is a symbol, written as sequences of letters, digits, and the underscore (_) and start with a lower-case letter.
  E.g., a, ai_is_fun, lit_l_1, $live\_outside$, $mimsy$, $sunny$.

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- A proposition or logical formula is either
  - an atomic proposition or
  - a compound proposition of the form

$\neg p$	"not p"	negation of p
$p \wedge q$	"p and q"	conjunction of p and q
$p \vee q$	"p or q"	disjunction of p and q
$p \rightarrow q$	"p implies q"	implication of q from p
$p \leftarrow q$	"p if q"	implication of p from q
$p \leftrightarrow q$	"p if and only if q"	equivalence of p and q
$p \oplus q$	"p XOR q"	exclusive-or of p and q

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- The operators $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftarrow$, $\leftrightarrow$, and $\oplus$ are logical connectives.

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- It can be extended to infinitely many propositions with infinite domains (using logical quantification)

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  If $\pi(a) = false$, atom a is false in the interpretation.
- Truth of a compound proposition in an interpretation is defined in terms of the truth of its components:

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftarrow q$	$p \leftrightarrow q$	$p \oplus q$
true	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*true*	*false*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*	*false*	*true*
false	*false*	*true*	*false*	*false*	*true*	*true*	*true*	*false*

- Propositions can have different truth values in different interpretations.

Models and Logical Consequence

- A model of a set of clauses is an interpretation in which all the clauses are *true*.
- If KB is a set of propositions, proposition g is a logical consequence of KB, written $KB \models g$, if g is *true* in every model of KB.
- That is, $KB \models g$ if there is no interpretation in which KB is *true* and g is *false*.

Simple Example

$$KB = \begin{cases} apple\_eaten \leftarrow bird\_eats\_apple. \\ light\_on \leftarrow night. \\ night. \end{cases}$$

	$apple\_eaten$	$bird\_eats\_apple$	$light\_on$	$night$	model of KB?
I_1	$true$	$true$	$true$	$true$	
I_2	$false$	$false$	$false$	$false$	
I_3	$true$	$true$	$false$	$false$	
I_4	$false$	$false$	$true$	$true$	
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Which of *apple_eaten*, *bird_eats_apple*, *light_on*, *night* logically follow from KB?

$KB \models light\_on$, $KB \models night$,

$KB \not\models apple\_eaten$, $KB \not\models bird\_eats\_apple$

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— You can interpret the answer with the meaning associated with the atoms.

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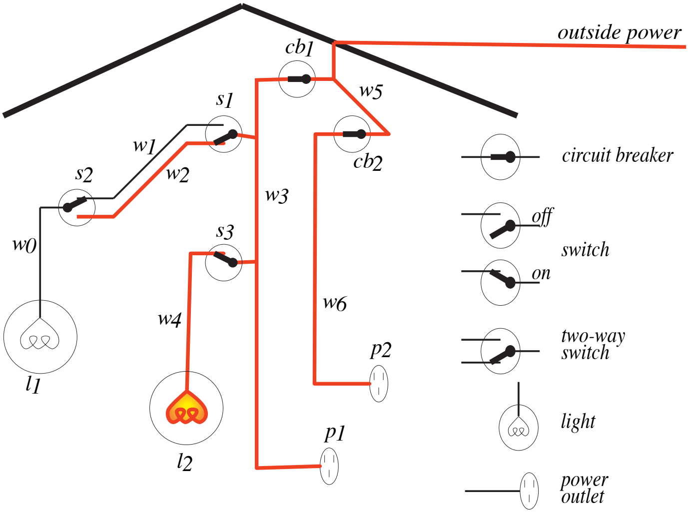
- If $KB \models g$ then g must be true in the intended interpretation.
- If $KB \not\models g$ then there is a model of KB in which g is false. This could be the intended interpretation.

Electrical Environment



Role of semantics

In computer:

$$light2\_broken \leftarrow power\_in\_w\_3 \wedge sw\_3\_up \wedge unlit\_light2.$$

$sw\_3\_up.$

$power\_in\_w\_3 \leftarrow power\_in\_p\_1.$

$unlit\_light2.$

$power\_in\_p\_1.$

In user's mind:

- $light2\_broken$: light #2 is broken
- sw_3_up: switch 3 is up
- $power\_in\_w\_3$: there is power in wire 3
- $unlit\_light2$: light #2 isn't lit
- $power\_in\_p\_1$: outlet p_1 has power

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Conclusion: $light2\_broken$

- The computer doesn't know the meaning of the symbols
- The user can interpret symbols using their meaning