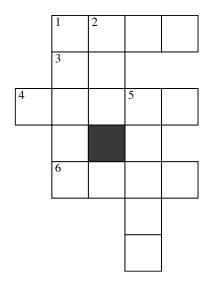
### Constraint satisfaction revisited

- A Constraint Satisfaction problem consists of:
  - a set of variables
  - ▶ a set of possible values, a domain for each variable
  - a set of constraints amongst subsets of the variables
- The aim is to find a set of assignments that satisfies all constraints, or to find all such assignments.



### Example: crossword puzzle



at, be, he, it, on, eta, hat, her, him, one, desk, dove, easy, else, help, kind, soon, this, dance, first, fuels, given, haste, loses, sense, sound, think, usage

### **Dual Representations**

#### Two ways to represent the crossword as a CSP

- First representation:
  - nodes represent word positions: 1-down...6-across
  - domains are the words
  - constraints specify that the letters on the intersections must be the same.
- Dual representation:
  - nodes represent the individual squares
  - domains are the letters
  - constraints specify that the words must fit



## Representations for image interpretation

- First representation:
  - nodes represent the chains and regions
  - domains are the scene objects
  - constraints correspond to the intersections and adjacency
- Dual representation:
  - nodes represent the intersections
  - domains are the intersection labels
  - constraints specify that the chains must have same marking

### Variable Elimination

 Idea: eliminate the variables one-by-one passing their constraints to their neighbours

#### Variable Elimination Algorithm:

- If there is only one variable, return the intersection of the (unary) constraints that contain it
- Select a variable X
- ullet Join the constraints in which X appears, forming constraint  $R_1$
- Project  $R_1$  onto its variables other than X, forming  $R_2$
- Replace all of the constraints in which  $X_i$  appears by  $R_2$
- Recursively solve the simplified problem, forming  $R_3$
- Return R<sub>1</sub> joined with R<sub>3</sub>

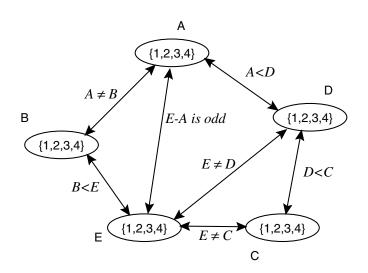


## Variable elimination (cont.)

- When there is a single variable remaining, if it has no values, the network was inconsistent.
- The variables are eliminated according to some elimination ordering
- Different elimination orderings result in different size intermediate constraints.

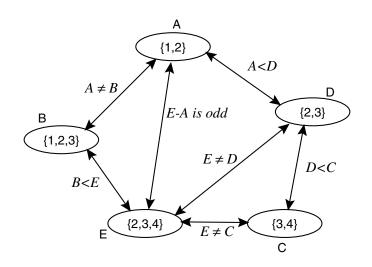


### Example network





### Example: arc-consistent network





## Example: eliminating C

r	1 : C 7	<i>E</i>	<i>C</i>	Ε		$r_2: C > D$	C	D
			3	2	-		3	2
			3	4			4	2
			4	2			4	3
			4	3		'		
$r_3 : r_1$	$\bowtie r2$	C	D	Ε		$r_4:\pi_{\{D,E\}}r_3$	D	Ε
		3	2	2	-		2	2
		3	2	4			2	3
		4	2	2			2	4
		4	2	3			3	2
		4	3	2			3	3
4			3	3		→ new constraint		

# Resulting network after eliminating C

