

At the end of the class you should be able to:

- recognize and represent constraint satisfaction problems
- count how big the search space is

Posing a Constraint Satisfaction Problem

A CSP is characterized by

- A set of **variables** V_1, V_2, \dots, V_n .
- Each variable V_i has an associated **domain** $dom(V_i)$ which specifies the set of possible values the variable can take. (We assume domains are finite.)
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- A **solution** to CSP is total assignment that satisfies all the constraints.

Example: Map colouring



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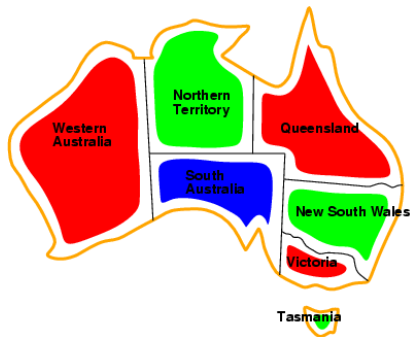
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Possible solution.



Simple Examples

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- Variables: A, B, C
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Example 3:

- Variables: A, B, C, D, E
- Domains: $\{1, 2, 3, 4\}$
- Constraints $A < B, B < C, C < D, D < E$

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- determine whether some property holds in all of the solutions

Example: scheduling activities

- **Variables:** A, B, C, D, E that represent the starting times of various activities.
- **Domains:** $dom(A) = \{1, 2, 3, 4\}$, $dom(B) = \{1, 2, 3, 4\}$,
 $dom(C) = \{1, 2, 3, 4\}$, $dom(D) = \{1, 2, 3, 4\}$,
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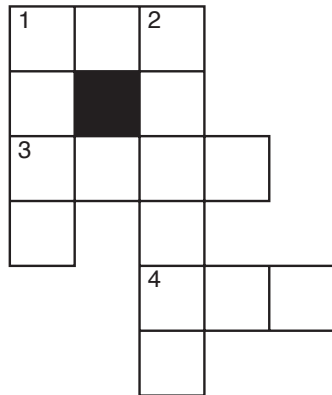
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- **Constraints:**

$$(B \neq 3) \wedge (C \neq 2) \wedge (A \neq B) \wedge (B \neq C) \wedge \\ (C < D) \wedge (A = D) \wedge (E < A) \wedge (E < B) \wedge \\ (E < C) \wedge (E < D) \wedge (B \neq D).$$

Example: Crossword Puzzle

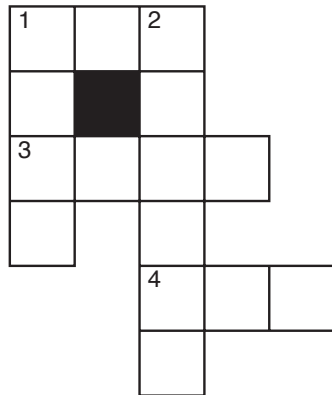


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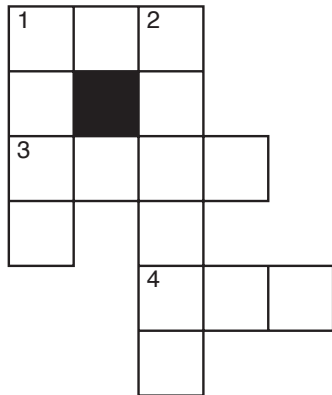


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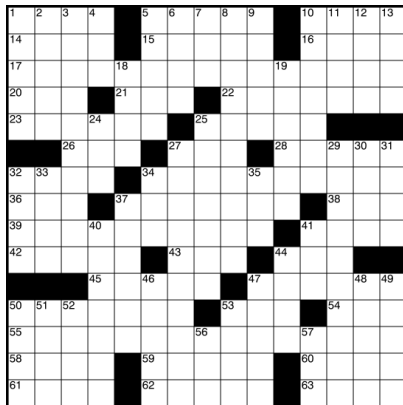


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Example: Crossword Puzzle



Suppose there are 10,000 words of each length (from 2 to 10).

- How many total assignments are there?

Example: Sudoku

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

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Hard and Soft Constraints

- Given a set of variables, assign a value to each variable that either
 - ▶ satisfies some set of constraints: **satisfiability problems** — “hard constraints”
 - ▶ minimizes some cost function, where each assignment of values to variables has some cost: **optimization problems** — “soft constraints”
- Many problems are a mix of hard and soft constraints (called constrained optimization problems).

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- 13 exam days, 52 timeslots
- 30,000 students take exams
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- Evening courses must have evening exams

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- Room capacities
- First-year exams on the last two days (Fall exams)
- Fourth-year exams on the last two days (Spring exams)

