## Learning Objectives

At the end of the class you should be able to:

- justify why depth-bounded search is useful
- demonstrate how iterative-deepening works for a particular problem
- demonstrate how depth-first branch-and-bound works for a particular problem


## Summary of Search Strategies

| Strategy | Frontier | Complete | Halts | Space |
| :--- | :--- | :--- | :--- | :--- |
| Depth-first w/o CP | Last added | No | No | Linear |
| Depth-first w CP | Last added | No | Yes | Linear |
| Depth-first w MPP | Last added | No | Yes | Exp |
| Breadth-first w/o MPP | First added | Yes | No | Exp |
| Breadth-first w MPP | First added | Yes | Yes | Exp |
| Best-first w/o MPP | Min $h(p)$ | No | No | Exp |
| Best-first w MPP | Min $h(p)$ | No | Yes | Exp |
| $A^{*}$ w/o MPP | Min $f(p)$ | Yes | No | Exp |
| $A^{*}$ w MPP | Min $f(p)$ | Yes | Yes | Exp |

Complete - if there a path to a goal, it can find one, even on infinite graphs.
Halts - on finite graph (perhaps with cycles).
Space - as a function of the length of current path
Assume graph satisfies the assumptions of $A^{*}$ proof + montonicity

## Bounded Depth-first search

- A bounded depth-first search takes a bound (cost or depth) and does not expand paths that exceed the bound.
- explores part of the search graph
- uses space linear in the depth of the search.
- How does this relate to other searches?
- How can this be extended to be complete?

Which shaded goal will a depth-bounded search find first?


## Iterative-deepening search

- Iterative-deepening search:
- Start with a bound $b=0$.
- Do a bounded depth-first search with bound $b$
- If a solution is found return that solution
- Otherwise increment $b$ and repeat.
- This will find the same first solution as what other method?
- How much space is used?
- What happens if there is no path to a goal?
- Surely recomputing paths is wasteful!!!


## Iterative Deepening Complexity

Complexity with solution at depth $k \&$ branching factor $b$ :

| level | breadth-first | iterative deepening | $\#$ nodes |
| :--- | :--- | :--- | :--- |
| 1 | 1 | $k$ | $b$ |
| 2 | 1 | $k-1$ | $b^{2}$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $k-1$ | 1 | 2 | $b^{k-1}$ |
| $k$ | 1 | 1 | $b^{k}$ |
| total | $\geq b^{k}$ | $\leq b^{k}\left(\frac{b}{b-1}\right)^{2}$ |  |

## Depth-first Branch-and-Bound

- combines depth-first search with heuristic information.
- finds optimal solution.
- most useful when there are multiple solutions, and we want an optimal one.
- uses the space of depth-first search.


## Depth-first Branch-and-Bound

Suppose we want to find a single optimal solution.

- Suppose bound is the cost of the lowest-cost path found to a goal so far.
- What if the search encounters a path $p$ such that $\operatorname{cost}(p)+h(p) \geq$ bound?
$p$ can be pruned.
- What can we do if a non-pruned path to a goal is found? bound can be set to the cost of $p$, and $p$ can be remembered as the best solution so far.
- Why should this use a depth-first search? Uses linear space.
- What can be guaranteed when the search completes? It has found an optimal solution.
- How should the bound be initialized?


## Depth-first Branch-and-Bound: Initializing Bound

- The bound can be initialized to $\infty$.
- The bound can be set to an estimate of the optimal path cost. After depth-first search terminates either:
- A solution was found.
- No solution was found, and no path was pruned
- No solution was found, and a path was pruned.

Which shaded goals will be best solutions so far?


