Learning Objectives

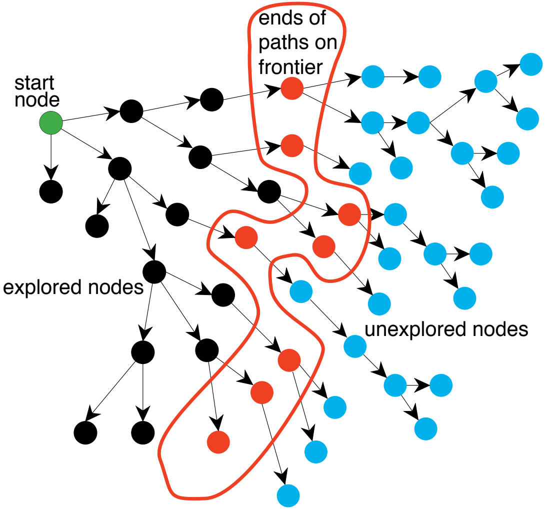
At the end of the class you should be able to:

- explain how a generic searching algorithm works
- demonstrate how depth-first search will work on a graph
- demonstrate how breadth-first search will work on a graph
- predict the space and time requirements for depth-first and breadth-first searches

Graph Searching

- Generic search algorithm: given a graph, start nodes, and goal nodes, incrementally explore paths from the start nodes.
- Maintain a frontier of paths from the start node that have been explored.
- As search proceeds, the frontier expands into the unexplored nodes until a goal node is encountered.
- The way in which the frontier is expanded defines the search strategy.

Problem Solving by Graph Searching



Graph Search Algorithm

Input: a graph,
a set of start nodes,
Boolean procedure $goal(n)$ that tests if n is a goal node.
$frontier := \{\langle s \rangle : s \text{ is a start node}\}$
while *frontier* is not empty:
  select and **remove** path $\langle n_0, \ldots, n_k \rangle$ from *frontier*
  if $goal(n_k)$
    return $\langle n_0, \ldots, n_k \rangle$
  for every neighbor n of n_k
    add $\langle n_0, \ldots, n_k, n \rangle$ to *frontier*
end while

Graph Search Algorithm

- Which value is selected from the frontier at each stage defines the search strategy.
- The neighbors define the graph.
- *goal* defines what is a solution.
- If more than one answer is required, the search can continue from the return.

Optimality Criteria

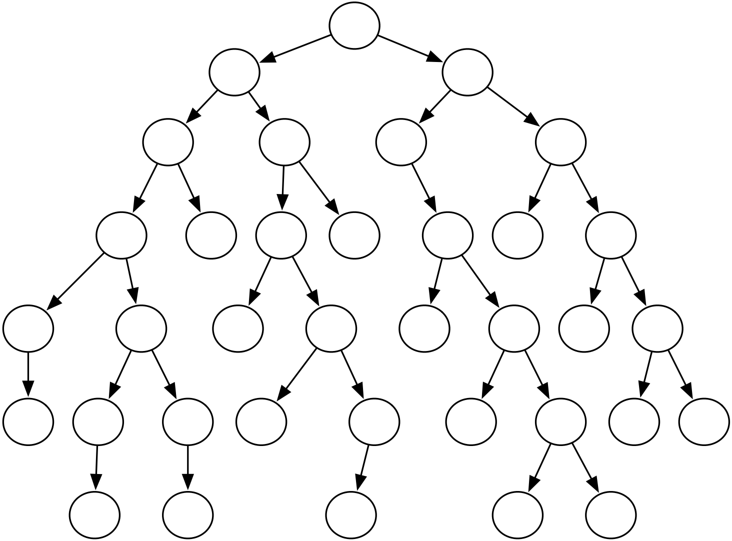
- Often we don't want any solution, but the best solution or optimal solution.
- Costs on arcs give costs on paths. We want the least-cost path to a goal.

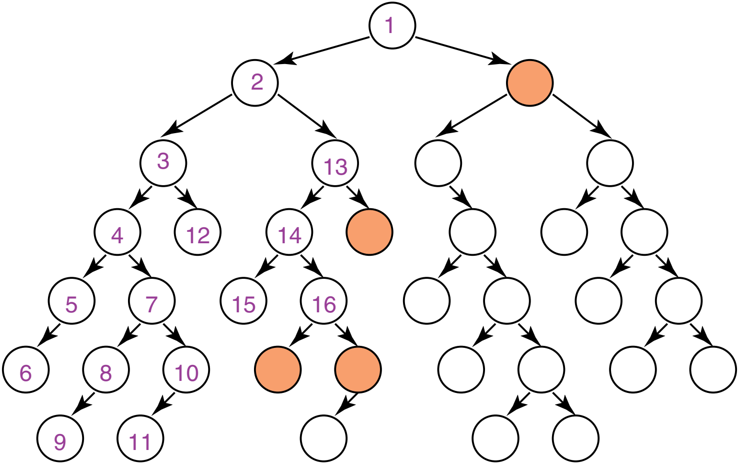
Depth-first Search

- Depth-first search treats the frontier as a stack
- It always selects one of the last elements added to the frontier.
- If the list of paths on the frontier is $[p_1, p_2, \ldots]$
    - p_1 is selected. Paths that extend p_1 are added to the front of the stack (in front of p_2).
    - p_2 is only selected when all paths from p_1 have been explored.

Illustrative Graph - Depth-first search

Start node is at the top.




1
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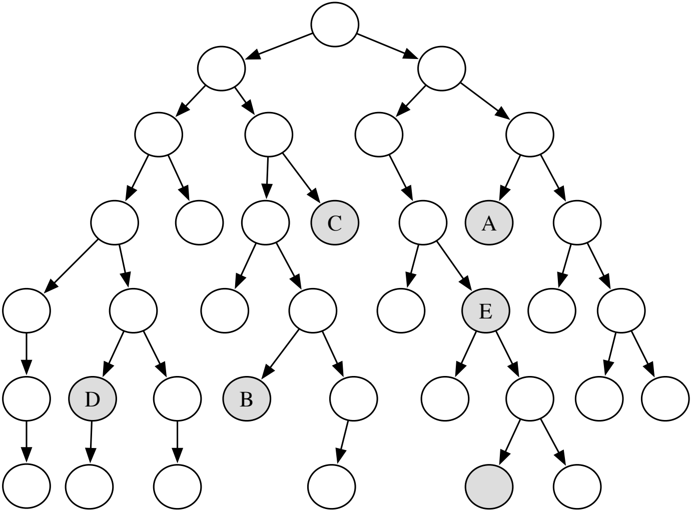
Which shaded goal will depth-first search find first?



Complexity of Depth-first Search

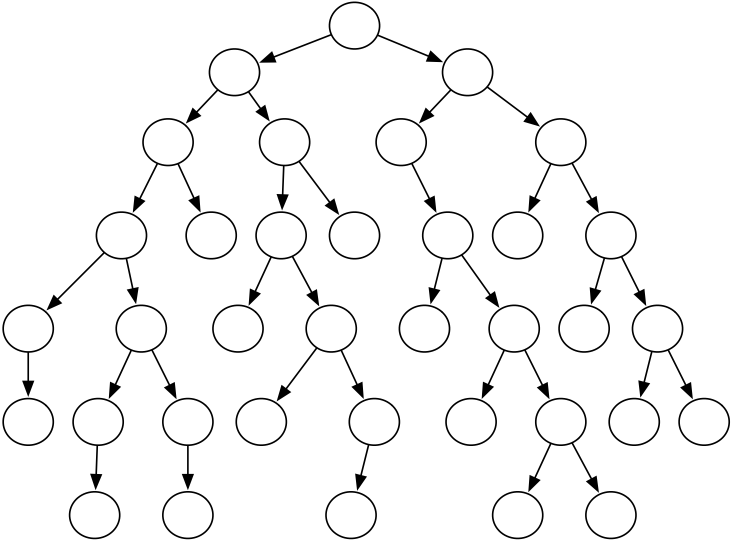
- Does depth-first search guarantee to find the path with fewest arcs?
- What happens on infinite graphs or on graphs with cycles if there is a solution?
- What is the time complexity as a function of length of the path selected?
- What is the space complexity as a function of length of the path selected?
- How does the goal affect the search?

Breadth-first Search

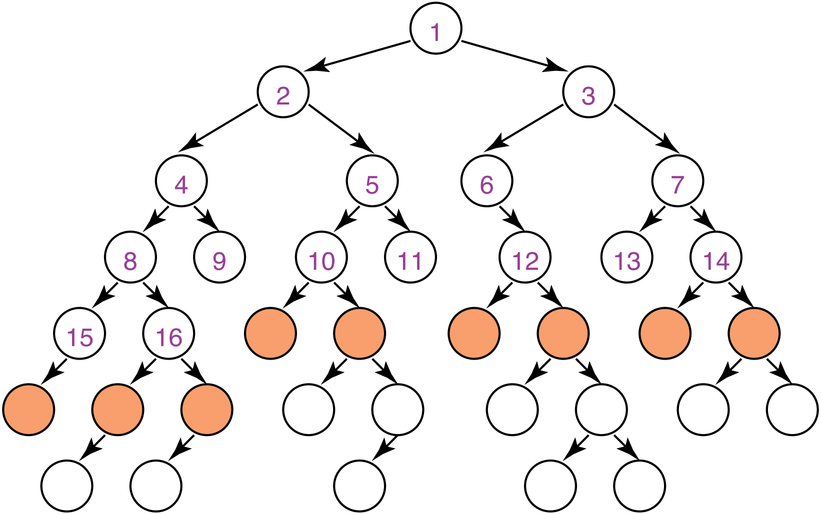
- Breadth-first search treats the frontier as a queue.
- It always selects one of the earliest elements added to the frontier.
- If the list of paths on the frontier is $[p_1, p_2, \ldots, p_r]$:
  - p_1 is selected. Its neighbors are added to the end of the queue, after p_r.
  - p_2 is selected next.

Illustrative Graph - Breadth-first search

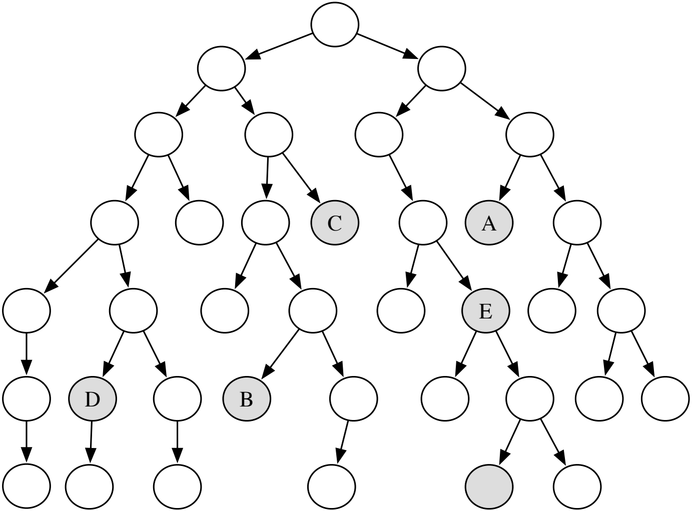
Start node is at the top.



Illustrative Graph — Breadth-first Search



Which shaded goal will breadth-first search find first?



Complexity of Breadth-first Search

- Does breadth-first search guarantee to find the path with fewest arcs?
- What happens on infinite graphs or on graphs with cycles if there is a solution?
- What is the time complexity as a function of the length of the path selected?
- What is the space complexity as a function of the length of the path selected?
- How does the goal affect the search?

Lowest-cost-first Search

- Sometimes there are costs associated with arcs. The cost of a path is the sum of the costs of its arcs.

$$cost(\langle n_0, \ldots, n_k \rangle) = \sum_{i=1}^{k} cost(\langle n_{i-1}, n_i \rangle)$$

  An optimal solution is one with minimum cost.
- At each stage, lowest-cost-first search selects a path on the frontier with lowest cost.
- The frontier is a priority queue ordered by path cost.
- The first path to a goal is a least-cost path to a goal node.
- When arc costs are equal $\Longrightarrow$ breadth-first search.

Summary of Search Strategies

Strategy	Frontier Selection	Complete	Halts	Space
Depth-first	Last node added	No	No	Linear
Breadth-first	First node added	Yes	No	Exp
Lowest-cost-first	Minimal $cost(p)$	Yes	No	Exp

Complete — guaranteed to find a solution if there is one (for graphs with finite number of neighbours, even on infinite graphs)

Halts — on finite graph (perhaps with cycles).

Space — as a function of the length of current path