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i.e. for all $x \in dom(X)$, $y, y' \in dom(Y)$, and $z \in dom(Z)$,

$$\begin{aligned} &P(X = x \mid Y = y \wedge Z = z) \\ &\quad = \quad P(X = x \mid Y = y' \wedge Z = z) \\ &\quad = \quad P(X = x \mid Z = z). \end{aligned}$$

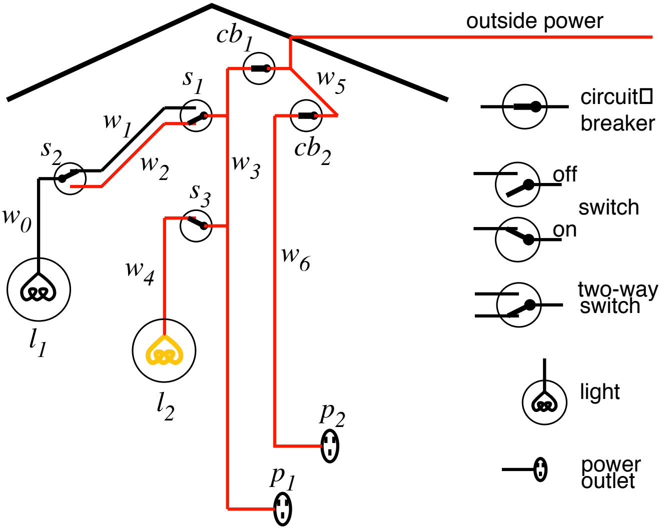
That is, knowledge of Y's value doesn't affect the belief in the value of X, given a value of Z.

Example

Consider a student writing an exam.
What are reasonable independences among the following?

- Whether the student works hard
- Whether the student is intelligent
- The student's answers on the exam
- The student's mark on an exam

Example domain (diagnostic assistant)



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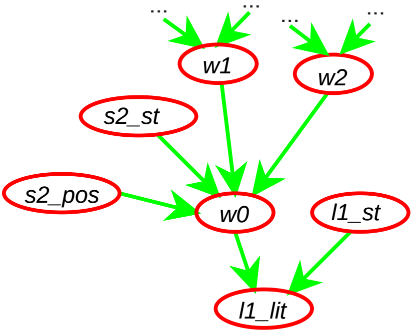
- Whether light $l1$ is lit is independent of the position of light switch $s2$ given what?
- Every other variable may be independent of whether light $l1$ is lit given whether there is power in wire w_0 and the status of light $l1$ (if it's *ok*, or if not, how it's broken).

Idea of belief networks

- $l1$ is lit ($L1\_lit$) depends only on the status of the light ($L1\_st$) and whether there is power in wire $w0$.
- In a belief network, $W0$ and $L1\_st$ are parents of $L1\_lit$.
- $W0$ depends only on



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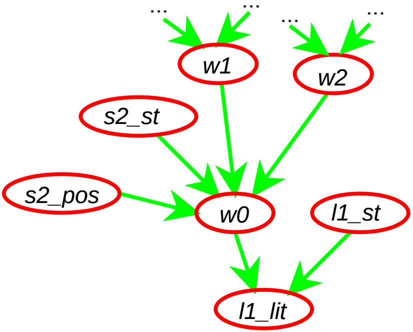

- $W0$ depends only on whether there is power in $w1$, whether there is power in $w2$, the position of switch $s2$ ($S2\_pos$), and the status of switch $s2$ ($S2\_st$).

Belief networks

- Totally order the variables of interest: $X_1, \ldots, X_n$
- Theorem of probability theory (chain rule): $P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i \mid X_1, \ldots, X_{i-1})$
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- The parents $parents(X_i)$ of X_i are those predecessors of X_i that render X_i independent of the other predecessors.
  That is, $parents(X_i) \subseteq X_1, \ldots, X_{i-1}$ and
  $P(X_i \mid parents(X_i)) = P(X_i \mid X_1, \ldots, X_{i-1})$
- So $P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i \mid parents(X_i))$
- A belief network is a graph: the nodes are random variables; there is an arc from the parents of each node into that node.

Example: fire alarm belief network

Variables:

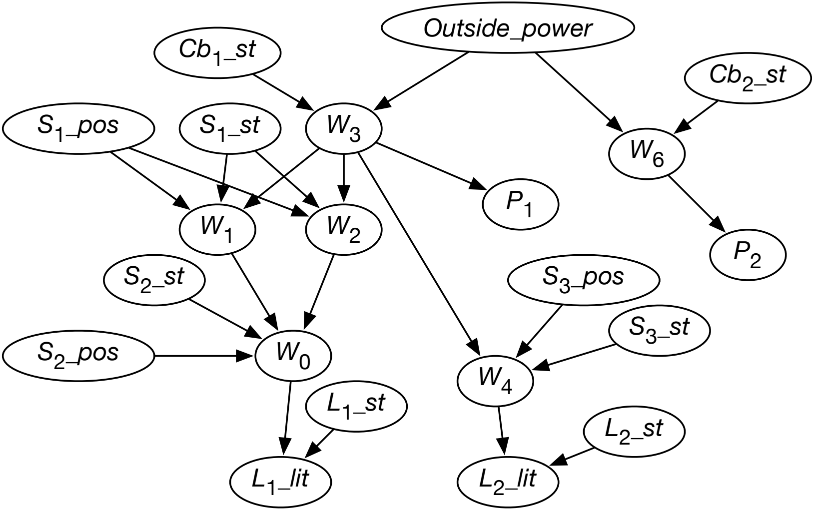
- Fire: there is a fire in the building
- Tampering: someone has been tampering with the fire alarm
- Smoke: what appears to be smoke is coming from an upstairs window
- Alarm: the fire alarm goes off
- Leaving: people are leaving the building *en masse*.
- Report: a colleague says that people are leaving the building *en masse*. (A noisy sensor for leaving.)

Components of a belief network

A belief network consists of:

- a directed acyclic graph with nodes labeled with random variables
- a domain for each random variable
- a set of conditional probability tables for each variable given its parents (including prior probabilities for nodes with no parents).

Example belief network



Example belief network (continued)

The belief network also specifies:

- The domain of the variables:
  $W_0, \ldots, W_6$ have domain $\{live, dead\}$
  S_1_pos, S_2_pos, and S_3_pos have domain $\{up, down\}$
  S_1_st has $\{ok, upside\_down, short, intermittent, broken\}$.
- Conditional probabilities, including:
  $P(W_1 = live \mid s_1\_pos = up \wedge S_1\_st = ok \wedge W_3 = live)$
  $P(W_1 = live \mid s_1\_pos = up \wedge S_1\_st = ok \wedge W_3 = dead)$
  $P(S_1\_pos = up)$
  $P(S_1\_st = upside\_down)$

Belief network summary

- A belief network is a directed acyclic graph (DAG) where nodes are random variables.
- The parents of a node n are those variables on which n directly depends.
- A belief network is automatically acyclic by construction.
- A belief network is a graphical representation of dependence and independence:
  - A variable is independent of its non-descendants given its parents.

Constructing belief networks

To represent a domain in a belief network, you need to consider:

- What are the relevant variables?
  - What will you observe?
  - What would you like to find out (query)?
  - What other features make the model simpler?
- What values should these variables take?
- What is the relationship between them? This should be expressed in terms of a directed graph, representing how each variable is generated from its predecessors.
- How does the value of each variable depend on its parents? This is expressed in terms of the conditional probabilities.