Learning Summary

- Given a task, use
  - data/experience
  - bias/background knowledge
  - measure of improvement or error

  to improve performance on the task.
- Representations for:
  - Data (e.g., discrete values, indicator functions)
  - Models (e.g., decision trees, linear functions, linear separators)
- A way to handle overfitting (e.g., trade-off model complexity and fit-to-data, cross validation).
- Search algorithm (usually local, myopic search) to find the best model that fits the data given the bias.

Learning Objectives - Reinforcement Learning

At the end of the class you should be able to:

- Explain the relationship between decision-theoretic planning (MDPs) and reinforcement learning
- Implement basic state-based reinforcement learning algorithms: Q-learning and SARSA
- Explain the explore-exploit dilemma and solutions
- Explain the difference between on-policy and off-policy reinforcement learning

Reinforcement Learning

What should an agent do given:

- Prior knowledge possible states of the world
  possible actions
- Observations current state of world
  immediate reward / punishment
- Goal act to maximize accumulated (discounted) reward

Like decision-theoretic planning, except model of dynamics and model of reward not given.

Reinforcement Learning Examples

- Game - reward winning, punish losing
- Dog - reward obedience, punish destructive behavior
- Robot - reward task completion, punish dangerous behavior

Experiences

- We assume there is a sequence of experiences:

$$state, action, reward, state, action, reward, \ldots.$$

- At any time it must decide whether to
  - explore to gain more knowledge
  - exploit knowledge it has already discovered

Why is reinforcement learning hard?

- What actions are responsible for a reward may have occurred a long time before the reward was received.
- The long-term effect of an action depend on what the agent will do in the future.
- The explore-exploit dilemma: at each time should the agent be greedy or inquisitive?

Reinforcement learning: main approaches

- search through a space of policies (controllers)
- learn a model consisting of state transition function $P(s'|a,s)$ and reward function $R(s,a,s')$; solve this an an MDP.
- learn $Q^*(s,a)$, use this to guide action.

Recall: Asynchronous VI for MDPs, storing $Q[s,a]$

(If we knew the model:)

Initialize $Q[S,A]$ arbitrarily
Repeat forever:

- Select state s, action a
- $Q[s,a] \leftarrow \sum_{s'} P(s'|s,a) \left(R(s,a,s') + \gamma \max_{a'} Q[s',a'] \right)$

Reinforcement Learning (Deterministic case)

- flat or modular or hierarchical
- explicit states or features or individuals and relations
- static or finite stage or indefinite stage or infinite stage
- fully observable or partially observable
- deterministic or stochastic dynamics
- goals or complex preferences
- single agent or multiple agents
- knowledge is given or knowledge is learned
- perfect rationality or bounded rationality

Experiential Asynchronous Value Iteration for Deterministic RL

initialize $Q[S, A]$ arbitrarily
observe current state s
repeat forever:
    select and carry out an action a
    observe reward r and state s'
        What do we know now?

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Temporal Differences

- Suppose we have a sequence of values:

  $$v_1, v_2, v_3, \ldots$$

  and want a running estimate of the average of the first k values:

  $$A_k = \frac{v_1 + \cdots + v_k}{k}$$

Temporal Differences (cont)

- Suppose we know A_{k-1} and a new value v_k arrives:

$$
\begin{aligned}
A_k &= \frac{v_1 + \cdots + v_{k-1} + v_k}{k} \\
&=
\end{aligned}
$$

Temporal Differences (cont)

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Let $\alpha_k = \frac{1}{k}$, then

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  Let $\alpha_k = \frac{1}{k}$, then

$$\begin{aligned} A_k &= (1 - \alpha_k) A_{k-1} + \alpha_k v_k \\ &= A_{k-1} + \alpha_k (v_k - A_{k-1}) \end{aligned}$$

  "TD formula"
- Often we use this update with α fixed.
- We can guarantee convergence to average if

$$\sum_{k=1}^{\infty} \alpha_k = \infty \text{ and } \sum_{k=1}^{\infty} \alpha_k^2 < \infty.$$

Q-learning

- Idea: store $Q[State, Action]$; update this as in asynchronous value iteration, but using experience (empirical probabilities and rewards).
- Suppose the agent has an experience $\langle s, a, r, s' \rangle$
- This provides one piece of data to update $Q[s, a]$.
- An experience $\langle s, a, r, s' \rangle$ provides a new estimate for the value of $Q^*(s, a)$:

which can be used in the TD formula giving:

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$$Q[s, a] \leftarrow Q[s, a] + \alpha \left(r + \gamma \max_{a'} Q[s', a'] - Q[s, a] \right)$$

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Properties of Q-learning

- Q-learning converges to an optimal policy, no matter what the agent does, as long as it tries each action in each state enough.
- But what should the agent do?
  - exploit: when in state s,
  - explore:

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- But what should the agent do?
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  - explore: select another action

Exploration Strategies

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  where $\tau > 0$ is the *temperature*.
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  where $\tau > 0$ is the *temperature*.
  Good actions are chosen more often than bad actions.
  τ defines how much a difference in Q-values maps to a difference in probability.
- "optimism in the face of uncertainty": initialize Q to values that encourage exploration.

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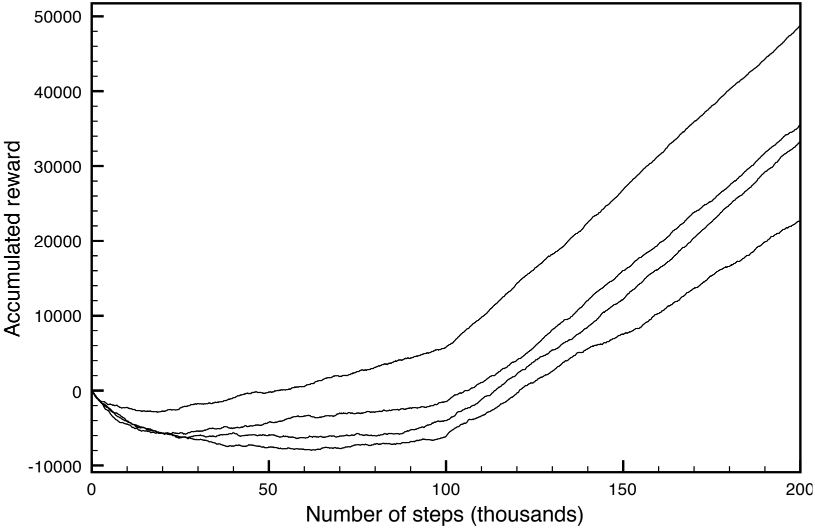
- It does one backup between each experience.
  - Is this appropriate for a robot interacting with the real world?
  - An agent can make better use of the data by
    — doing multi-step backups
    — building a model, and using MDP methods to determine optimal policy.
- It learns separately for each state.

Evaluating Reinforcement Learning Algorithms



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- On-policy learning learns the value of the policy being followed.
  e.g., act greedily 80% of the time and act randomly 20% of the time
- Why?

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  e.g., act greedily 80% of the time and act randomly 20% of the time
- Why? If the agent is actually going to explore, it may be better to optimize the actual policy it is going to do.
- SARSA uses the experience $\langle s, a, r, s', a' \rangle$ to update $Q[s, a]$.

SARSA

initialize $Q[S, A]$ arbitrarily
observe current state s
select action a using a policy based on Q
repeat forever:
    carry out action a
    observe reward r and state s'
    select action a' using a policy based on Q
    $Q[s, a] \leftarrow$

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Reinforcement Learning with Features

- Usually we don't want to reason in terms of states, but in terms of features.
- In state-based methods, information about one state cannot be used by similar states.
- If there are too many parameters to learn, it takes too long.
- Idea: Express the value function as a function of the features. Most typical is a linear function of the features.

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Gradient descent

To find a (local) minimum of a real-valued function $f(x)$:

- assign an arbitrary value to x
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for each x_i

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- repeat:

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$$x_i \leftarrow x_i - \eta \frac{\partial f}{\partial x_i}$$

Linear Regression

- A linear function of variables $x_1, \ldots, x_n$ is of the form

$$f^{\overline{w}}(x_1, \ldots, x_n) = w_0 + w_1 x_1 + \cdots + w_n x_n$$

  $\overline{w} = \langle w_0, w_1, \ldots, w_n \rangle$ are weights. (Let $x_0 = 1$).
- Given a set E of examples.
  Example e has input $x_i = e_i$ for each i and observed value, o_e:

$$Error_E(\overline{w}) = \sum_{e \in E} (o_e - f^{\overline{w}}(e_1, \ldots, e_n))^2$$

- Minimizing the error using gradient descent, each example should update w_i using:

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Gradient Descent for Linear Regression

Given E: set of examples over n features
each example e has inputs $(e_1, \ldots, e_n)$ and output o_e:
Assign weights $\overline{w} = \langle w_0, \ldots, w_n \rangle$ arbitrarily
repeat:
For each example e in E:
let $\delta = o_e - f^{\overline{w}}(e_1, \ldots, e_n)$
For each weight w_i:
$w_i \leftarrow w_i + \eta \delta e_i$

SARSA with linear function approximation

- One step backup provides the examples that can be used in a linear regression.
- Suppose $F_1, \ldots, F_n$ are the features of the state and the action.
- So $Q_{\overline{w}}(s,a) = w_0 + w_1 F_1(s,a) + \cdots + w_n F_n(s,a)$
- An experience $\langle s, a, r, s', a' \rangle$ provides the "example":
  - old predicted value:
  - new "observed" value:

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- An experience $\langle s, a, r, s', a' \rangle$ provides the "example":
  - old predicted value: $Q_{\overline{w}}(s, a)$
  - new "observed" value: $r + \gamma Q_{\overline{w}}(s', a')$

SARSA with linear function approximation

Given γ:discount factor; η:step size
Assign weights $\overline{w} = \langle w_0, \ldots, w_n \rangle$ arbitrarily
observe current state s
select action a
repeat forever:
    carry out action a
    observe reward r and state s'
    select action a' (using a policy based on $Q_{\overline{w}}$)
    let $\delta = r + \gamma Q_{\overline{w}}(s', a') - Q_{\overline{w}}(s, a)$
    For $i = 0$ to n
        $w_i \leftarrow w_i + \eta \delta F_i(s, a)$
    $s \leftarrow s'$
    $a \leftarrow a'$

Example Features

- $F_1(s,a) = 1$ if a goes from state s into a monster location and is 0 otherwise.
- $F_2(s,a) = 1$ if a goes into a wall, is 0 otherwise.
- $F_3(s,a) = 1$ if a goes toward a prize.
- $F_4(s,a) = 1$ if the agent is damaged in state s and action a takes it toward the repair station.
- $F_5(s,a) = 1$ if the agent is damaged and action a goes into a monster location.
- $F_6(s,a) = 1$ if the agent is damaged.
- $F_7(s,a) = 1$ if the agent is not damaged.
- $F_8(s,a) = 1$ if the agent is damaged and there is a prize in direction a.
- $F_9(s,a) = 1$ if the agent is not damaged and there is a prize in direction a.

Example Features

- $F_{10}(s, a)$ is the distance from the left wall if there is a prize at location P_0, and is 0 otherwise.
- $F_{11}(s, a)$ has the value $4 - x$, where x is the horizontal position of state s if there is a prize at location P_0; otherwise is 0.
- $F_{12}(s, a)$ to $F_{29}(s, a)$ are like F_{10} and F_{11} for different combinations of the prize location and the distance from each of the four walls.
  For the case where the prize is at location P_0, the y-distance could take into account the wall.

Model-based Reinforcement Learning

- Model-based reinforcement learning uses the experiences in a more effective manner.
- It is used when collecting experiences is expensive (e.g., in a robot or an online game); an agent can do lots of computation between each experience.
- Idea: learn the MDP and interleave acting and planning.
- After each experience, update probabilities and the reward, then do some steps of asynchronous value iteration.

Model-based learner

Data Structures: $Q[S, A]$, $T[S, A, S]$, $C[S, A]$, $R[S, A]$
Assign Q, R arbitrarily, $C = 0$, $T = 0$
observe current state s
repeat forever:

select and carry out action a
observe reward r and state s'
$T[s, a, s'] \leftarrow T[s, a, s'] + 1$
$C[s, a] \leftarrow C[s, a] + 1$
$R[s, a] \leftarrow R[s, a] + (r - R[s, a])/C[s, a]$
repeat for a while:

select state s_1, action a_1

$$Q[s_1, a_1] \leftarrow R[s_1, a_1] + \sum_{s_2} \frac{T[s_1, a_1, s_2]}{C[s_1, a_1]} \left(\gamma \max_{a_2} Q[s_2, a_2] \right)$$

$s \leftarrow s'$

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What goes wrong with this?

Evolutionary Algorithms

- Idea:
  - maintain a population of controllers
  - evaluate each controller by running it in the environment
  - at each generation, the best controllers are combined to form a new population of controllers

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Evolutionary Algorithms

- Idea:
  - maintain a population of controllers
  - evaluate each controller by running it in the environment
  - at each generation, the best controllers are combined to form a new population of controllers
- If there are n states and m actions, there are m^n policies.
- Experiences are used wastefully: only used to judge the whole controller. They don't learn after every step.
- Performance is very sensitive to representation of controller.