# Goals and Preferences

Alice ... went on "Would you please tell me, please, which way I ought to go from here?"
"That depends a good deal on where you want to get to," said the Cat.
"I don't much care where —" said Alice.
"Then it doesn't matter which way you go," said the Cat.

*Lewis Carroll, 1832–1898*
*Alice's Adventures in Wonderland, 1865*
*Chapter 6*

# Learning Objectives

At the end of the class you should be able to:

- justify the use and semantics of utility
- estimate the utility of an outcome
- build a decision network for a domain
- compute the optimal policy of a decision network

# Preferences

- Actions result in outcomes
- Agents have preferences over outcomes
- A rational agent will do the action that has the best outcome for them
- Sometimes agents don't know the outcomes of the actions, but they still need to compare actions
- Agents have to act.
  (Doing nothing is (often) an action).

# Preferences Over Outcomes

If $o_1$ and $o_2$ are outcomes

- $o_1 \succeq o_2$ means $o_1$ is at least as desirable as $o_2$.
- $o_1 \sim o_2$ means $o_1 \succeq o_2$ and $o_2 \succeq o_1$.
- $o_1 \succ o_2$ means $o_1 \succeq o_2$ and $o_2 \not\succeq o_1$

# Lotteries

- An agent may not know the outcomes of their actions, but only have a probability distribution of the outcomes.
- A lottery is a probability distribution over outcomes. It is written

$$[p_1 : o_1, p_2 : o_2, \ldots, p_k : o_k]$$

  where the $o_i$ are outcomes and $p_i \geq 0$ such that

$$\sum_i p_i = 1$$

  The lottery specifies that outcome $o_i$ occurs with probability $p_i$.
- When we talk about outcomes, we will include lotteries.

# Properties of Preferences

- Completeness: Agents have to act, so they must have preferences:

$$\forall o_1 \forall o_2 \; o_1 \succeq o_2 \text{ or } o_2 \succeq o_1$$

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- **Transitivity:** Preferences must be transitive:

$$\text{if } o_1 \succeq o_2 \text{ and } o_2 \succ o_3 \text{ then } o_1 \succ o_3$$

(Similarly for other mixtures of $\succ$ and $\succeq$.)

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(Similarly for other mixtures of $\succ$ and $\succeq$.)
Rationale: otherwise $o_1 \succeq o_2$ and $o_2 \succ o_3$ and $o_3 \succeq o_1$.
If they are prepared to pay to get $o_2$ instead of $o_3$,
and are happy to have $o_1$ instead of $o_2$,
and are happy to have $o_3$ instead of $o_1$
$\longrightarrow$ money pump.

# Properties of Preferences (cont.)

Monotonicity: An agent prefers a larger chance of getting a better outcome than a smaller chance:

- If $o_1 \succ o_2$ and $p > q$ then

$$[p : o_1, 1 - p : o_2] \succ [q : o_1, 1 - q : o_2]$$

# Consequence of axioms

- Suppose $o_1 \succ o_2$ and $o_2 \succ o_3$. Consider whether the agent would prefer
  - $o_2$
  - the lottery $[p : o_1, 1 - p : o_3]$

  for different values of $p \in [0, 1]$.
- Plot which one is preferred as a function of $p$:

# Properties of Preferences (cont.)

Continuity: Suppose $o_1 \succ o_2$ and $o_2 \succ o_3$, then there exists a $p \in [0,1]$ such that

$$o_2 \sim [p : o_1, 1-p : o_3]$$

# Properties of Preferences (cont.)

**Decomposability:** (no fun in gambling). An agent is indifferent between lotteries that have same probabilities and outcomes. This includes lotteries over lotteries. For example:

$$[p : o_1, 1 - p : [q : o_2, 1 - q : o_3]]$$
$$\sim \quad [p : o_1, (1 - p)q : o_2, (1 - p)(1 - q) : o_3]$$

# Properties of Preferences (cont.)

**Substitutability:** if $o_1 \sim o_2$ then the agent is indifferent between lotteries that only differ by $o_1$ and $o_2$:

$$[p : o_1, 1 - p : o_3] \sim [p : o_2, 1 - p : o_3]$$

# Alternative Axiom for Substitutability

Substitutability: if $o_1 \succeq o_2$ then the agent weakly prefers lotteries that contain $o_1$ instead of $o_2$, everything else being equal.
That is, for any number $p$ and outcome $o_3$:

$$[p : o_1, (1-p) : o_3] \succeq [p : o_2, (1-p) : o_3]$$

# What we would like

- We would like a measure of preference that can be combined with probabilities. So that

$$\begin{aligned} & value([p : o_1, 1-p : o_2]) \\ = \ & p \times value(o_1) + (1-p) \times value(o_2) \end{aligned}$$

- Money does not act like this.
  What would you prefer

$$\$1,000,000 \text{ or } [0.5 : \$0, 0.5 : \$2,000,000]?$$

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  What would you prefer
$$\$1,000,000 \text{ or } [0.5 : \$0, 0.5 : \$2,000,000]?$$
- It may seem that preferences are too complex and muti-faceted to be represented by single numbers.

# Theorem

If preferences follow the preceding properties, then preferences can be measured by a function

$$utility : outcomes \rightarrow [0, 1]$$

such that

- $o_1 \succeq o_2$ if and only if $utility(o_1) \geq utility(o2)$.
- Utilities are linear with probabilities:

$$\begin{aligned} & utility([p_1 : o_1, p_2 : o_2, \ldots, p_k : o_k]) \\ = & \sum_{i=1}^{k} p_i \times utility(o_i) \end{aligned}$$

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- Otherwise, suppose the best outcome is *best* and the worst outcome is *worst*.
- For any outcome $o_i$, define $utility(o_i)$ to be the number $u_i$ such that

$$o_i \sim [u_i : best, 1 - u_i : worst]$$

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This exists by the Continuity property.

# Proof (cont.)

- Suppose $o_1 \succeq o_2$ and $utility(o_i) = u_i$, then by Substitutability,

$$[u_1 : best, 1 - u_1 : worst]$$
$$\succeq$$

# Proof (cont.)

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  Which, by completeness and monotonicity implies $u_1 \geq u_2$.

# Proof (cont.)

- Suppose $p = utility([p_1 : o_1, p_2 : o_2, \ldots, p_k : o_k])$.
- Suppose $utility(o_i) = u_i$. We know:

$$o_i \sim [u_i : best, 1 - u_i : worst]$$

- By substitutability, we can replace each $o_i$ by $[u_i : best, 1 - u_i : worst]$, so

$$\begin{aligned} p = utility( \quad [ \quad & p_1 : [u_1 : best, 1 - u_1 : worst] \\ & \ldots \\ & p_k : [u_k : best, 1 - u_k : worst]]) \end{aligned}$$

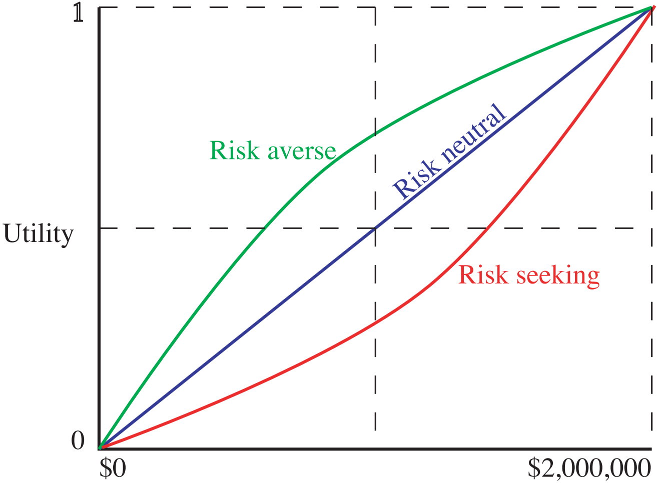
- By decomposability, this is equivalent to:

$$p = utility(\;[\;p_1 u_1 + \cdots + p_k u_k \\ : best, \\ p_1(1-u_1) + \cdots + p_k(1-u_k) \\ : worst]])$$

- Thus, by definition of utility,

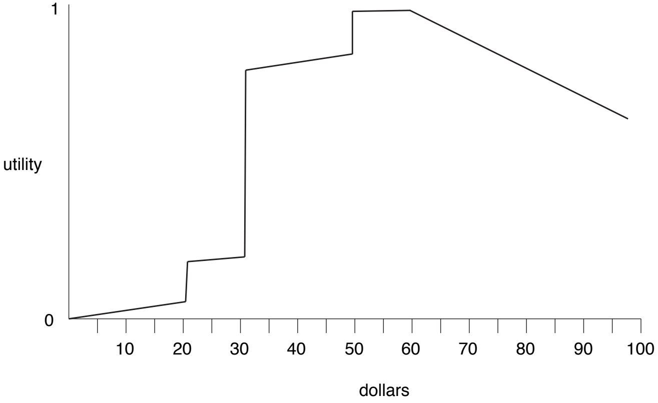
$$p = p_1 \times u_1 + \cdots + p_k \times u_k$$

# Utility as a function of money



# Possible utility as a function of money

Someone who really wants a toy worth \$30, but who would also like one worth \$20:



# Factored Representation of Utility

- Suppose the outcomes can be described in terms of features $X_1, \ldots, X_n$.
- An additive utility is one that can be decomposed into set of factors:

$$u(X_1, \ldots, X_n) = f_1(X_1) + \cdots + f_n(X_n).$$

This assumes additive independence.

- Strong assumption: contribution of each feature doesn't depend on other features.
- Many ways to represent the same utility:
— a number can be added to one factor as long as it is subtracted from others.

# Additive Utility

- An additive utility has a canonical representation:

$$u(X_1, \ldots, X_n) = w_1 \times u_1(X_1) + \cdots + w_n \times u_n(X_n).$$

- If $best_i$ is the best value of $X_i$, $u_i(X_i{=}best_i) = 1$.
  If $worst_i$ is the worst value of $X_i$, $u_i(X_i{=}worst_i) = 0$.
- $w_i$ are weights, $\sum_i w_i = 1$.
  The weights reflect the relative importance of features.
- We can determine weights by comparing outcomes.

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$$w_1 = u(best_1, x_2, \dots, x_n) - u(worst_1, x_2, \dots, x_n).$$

  for any values $x_2, \dots, x_n$ of $X_2, \dots, X_n$.

# General Setup for Additive Utility

Suppose there are:

- multiple users
- multiple alternatives to choose among, e.g., *hotel*1,...
- multiple criteria upon which to judge, e.g., *rate*, *location*
- utility is a function of

# General Setup for Additive Utility

Suppose there are:

- multiple users
- multiple alternatives to choose among, e.g., $hotel1, \ldots$
- multiple criteria upon which to judge, e.g., $rate$, $location$
- utility is a function of users and alternatives
- $fact(crit, alt)$ is the fact about the domain value of criteria $crit$ for alternative $alt$.
  E.g., $fact(rate, hotel1)$ is the room rate for hotel#1, which is \$125 per night.
- $score(val, user, crit)$ gives the score of the domain value for user on criteria $crit$.

$$
\begin{aligned}
utility(user, alt) = \sum_{crit} \; & weight(user, crit) \times \\
& score(fact(crit, alt), user, crit)
\end{aligned}
$$

for $user$, alternative $alt$, criteria $crit$

# Complements and Substitutes

- Often additive independence is not a good assumption.
- Values $x_1$ of feature $X_1$ and $x_2$ of feature $X_2$ are **complements** if having both is better than the sum of the two.
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- Example: on a holiday
  - A trip to a location 3 hours North on day 3
  - The return trip for the same day.

# Generalized Additive Utility

- A generalized additive utility can be written as a sum of factors:

$$u(X_1, \ldots, X_n) = f_1(\overline{X_1}) + \cdots + f_k(\overline{X_k})$$

  where $\overline{X_i} \subseteq \{X_1, \ldots, X_n\}$.
- An intuitive canonical representation is difficult to find.
- It can represent complements and substitutes.

# Utility and time

- Would you prefer \$1000 today or \$1000 next year?
- What price would you pay now to have an eternity of happiness?
- How can you trade off pleasures today with pleasures in the future?

# Pascal's Wager (1670)

Decide whether to believe in God.

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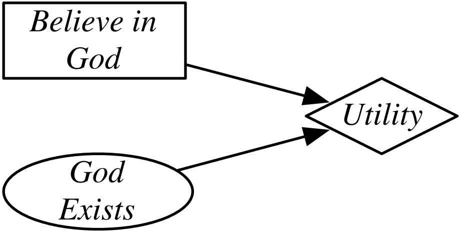

# Utility and time

- How would you compare the following sequences of rewards (per week):

  A: \$1000000, \$0, \$0, \$0, \$0, \$0,...
  B: \$1000, \$1000, \$1000, \$1000, \$1000,...
  C: \$1000, \$0, \$0, \$0, \$0,...
  D: \$1, \$1, \$1, \$1, \$1,...
  E: \$1, \$2, \$3, \$4, \$5,...

# Rewards and Values

Suppose the agent receives a sequence of rewards $r_1, r_2, r_3, r_4, \ldots$ in time. What utility should be assigned? "Return" or "value"

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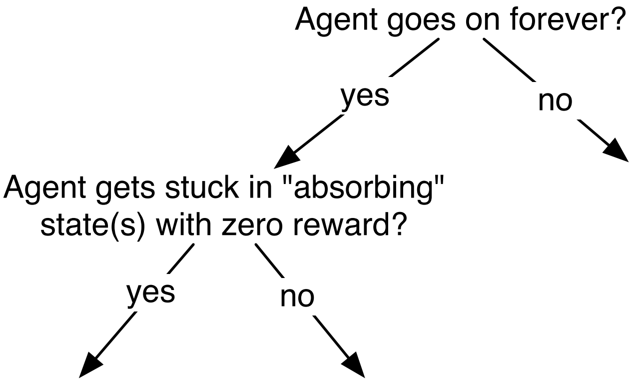
- total reward $V = \sum_{i=1}^{\infty} r_i$
- average reward $V = \lim_{n \to \infty} (r_1 + \cdots + r_n)/n$

# Average vs Accumulated Rewards



# Rewards and Values

Suppose the agent receives a sequence of rewards $r_1, r_2, r_3, r_4, \ldots$ in time.

- discounted return $V = r_1 + \gamma r_2 + \gamma^2 r_3 + \gamma^3 r_4 + \cdots$
  $\gamma$ is the discount factor $0 \leq \gamma \leq 1$.

# Properties of the Discounted Rewards

- The discounted return for rewards $r_1, r_2, r_3, r_4, \ldots$ is

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$$V_t = r_t + \gamma V_{t+1}$$
- How is the infinite future valued compared to immediate rewards?
$1 + \gamma + \gamma^2 + \gamma^3 + \cdots = 1/(1-\gamma)$
Therefore $\dfrac{\text{minimum reward}}{1-\gamma} \leq V_t \leq \dfrac{\text{maximum reward}}{1-\gamma}$
- We can approximate $V$ with the first $k$ terms, with error:
$$V - (r_1 + \gamma r_2 + \cdots + \gamma^{k-1} r_k) = \gamma^k V_{k+1}$$

# Allais Paradox (1953)

What would you prefer:

A: $\$1m$ — one million dollars

B: lottery $[0.10 : \$2.5m, 0.89 : \$1m, 0.01 : \$0]$

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C: lottery $[0.11 : \$1m, 0.89 : \$0]$

D: lottery $[0.10 : \$2.5m, 0.9 : \$0]$

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What would you prefer:

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It is inconsistent with the axioms of preferences to have $A \succ B$ and $D \succ C$.

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A,C: lottery $[0.11 : \$1m, 0.89 : X]$

B,D: lottery $[0.10 : \$2.5m, 0.01 : \$0, 0.89 : X]$

# Framing Effects [Tversky and Kahneman]

- A disease is expected to kill 600 people. Two alternative programs have been proposed:

  Program A: 200 people will be saved

  Program B: probability $1/3$: 600 people will be saved
  probability $2/3$: no one will be saved

  Which program would you favor?

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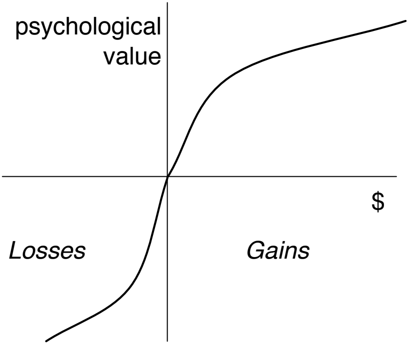
Tversky and Kahneman: 72% chose A over B.
22% chose C over D.

# Prospect Theory




- In mixed gambles, loss aversion causes extreme risk-averse choices
- In bad choices, diminishing responsibility causes risk seeking.

# Reference Points

Consider Anthony and Betty:

- Anthony's current wealth is \$1 million.
- Betty's current wealth is \$4 million.

They are both offered the choice between a gamble and a sure thing:

- Gamble: equal chance to end up owning \$1 million or \$4 million.
- Sure Thing: own \$2 million

What does expected utility theory predict?

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What does expected utility theory predict?
What does prospect theory predict?

[From D. Kahneman, *Thinking, Fast and Slow*, 2011, pp. 275-276.]

# Framing Effects

What do you think of Alan and Ben:

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[From D. Kahneman, Thinking Fast and Slow, 2011, p. 82]

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- Suppose you had \$50 in your pocket to buy tickets. When you got to the theatre, you had lost the \$50. You have your credit card and can buy equivalent tickets for \$50. Do you buy the tickets on your credit card?

[From R.M. Dawes, Rational Choice in an Uncertain World, 1988.]

# The Ellsberg Paradox

Two bags:

- **Bag 1** 40 white chips, 30 yellow chips, 30 green chips
- **Bag 2** 40 white chips, 60 chips that are yellow or green

What do you prefer:

- A: Receive \$1m if a white or yellow chip is drawn from bag 1
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However $A$ and $D$ should give same outcome, no matter what the proportion in Bag 2.

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What if there is no "best" outcome?
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- Suppose utilities are unbounded.
- Then for any outcome $o_i$ there is an outcome $o_{i+1}$ such that $u(o_{i+1}) > 2u(o_i)$.

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- What will eventually happen?

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- The "predictor" has put \$1m in box 2 if he thinks you will take box 2 and \$0 in box 2 if he thinks you will take both.
- The predictor has been correct in previous predictions.
- Do you take both boxes or just box 2?