Learning Objectives

At the end of the class you should be able to:

- identify a supervised learning problem
- characterize how the prediction is a function of the error measure
- avoid mixing the training and test sets

Supervised Learning

Given:

- a set of inputs features $X_1, \ldots, X_n$
- a set of target features $Y_1, \ldots, Y_k$
- a set of training examples where the values for the input features and the target features are given for each example
- a new example, where only the values for the input features are given

predict the values for the target features for the new example.

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- classification when the Y_i are discrete
- regression when the Y_i are continuous

Example Data Representations

A travel agent wants to predict the preferred length of a trip, which can be from 1 to 6 days. (No input features).

Two representations of the same data:

— Y is the length of trip chosen.

— Each Y_i is an indicator variable that has value 1 if the chosen length is i, and is 0 otherwise.

Example	Y
e_1	1
e_2	6
e_3	6
e_4	2
e_5	1

Example	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
e_1	1	0	0	0	0	0
e_2	0	0	0	0	0	1
e_3	0	0	0	0	0	1
e_4	0	1	0	0	0	0
e_5	1	0	0	0	0	0

What is a prediction?

Evaluating Predictions

Suppose we want to make a prediction of a value for a target feature on example e:

- o_e is the observed value of target feature on example e.
- p_e is the predicted value of target feature on example e.
- The error of the prediction is a measure of how close p_e is to o_e.
- There are many possible errors that could be measured.

Sometimes p_e can be a real number even though o_e can only have a few values.

Measures of error

E is the set of examples, with single target feature. For $e \in E$, o_e is observed value and p_e is predicted value:

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- number wrong: $L_0(E) = \#\{e : o_e \neq p_e\}$
- A cost-based error takes into account costs of errors.

Measures of error (cont.)

With binary feature: $o_e \in \{0, 1\}$:

- likelihood of the data

$$\prod_{e \in E} p_e^{o_e}(1 - p_e)^{(1-o_e)}$$

Measures of error (cont.)

With binary feature: $o_e \in \{0, 1\}$:

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- log likelihood

$$\sum_{e \in E} (o_e \log p_e + (1 - o_e) \log(1 - p_e))$$

is negative of number of bits to encode the data given a code based on p_e.

Information theory overview

- A bit is a binary digit.
- 1 bit can distinguish 2 items
- k bits can distinguish 2^k items
- n items can be distinguished using $\log_2 n$ bits
- Can we do better?

Information and Probability

Consider a code to distinguish elements of $\{a, b, c, d\}$ with

$$P(a) = \frac{1}{2}, P(b) = \frac{1}{4}, P(c) = \frac{1}{8}, P(d) = \frac{1}{8}$$

Consider the code:

a	0	b	10	c	110	d	111

This code uses 1 to 3 bits. On average, it uses

$$\begin{aligned} & P(a) \times 1 + P(b) \times 2 + P(c) \times 3 + P(d) \times 3 \\ = \ & \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{3}{8} = 1\frac{3}{4} \text{ bits.} \end{aligned}$$

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The code 0111110010100 represents string *adcabba*

Information Content

- To identify x, we need $-\log_2 P(x)$ bits.
- Give a distribution over a set, to a identify a member, the expected number of bits

$$\sum_x -P(x) \times \log_2 P(x).$$

is the information content or entropy of the distribution.

- The expected number of bits it takes to describe a distribution given evidence e:

$$I(e) = \sum_x -P(x|e) \times \log_2 P(x|e).$$

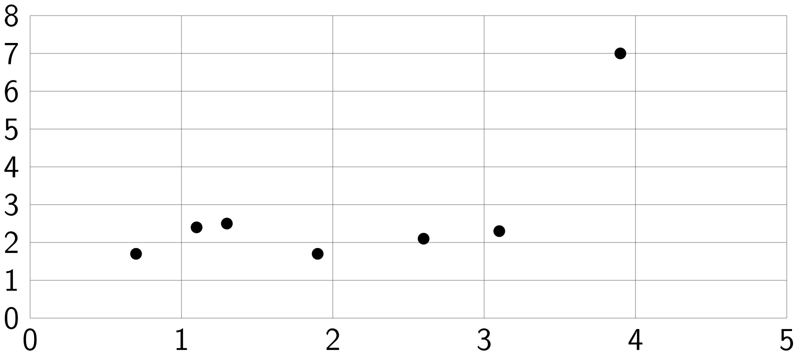
Information Gain

Given a test that can distinguish the cases where α is true from the cases where α is false, the information gain from this test is:

$$I(true) - (P(\alpha) \times I(\alpha) + P(\neg\alpha) \times I(\neg\alpha)).$$

- $I(true)$ is the expected number of bits needed before the test
- $P(\alpha) \times I(\alpha) + P(\neg\alpha) \times I(\neg\alpha)$ is the expected number of bits after the test.

Linear Predictions



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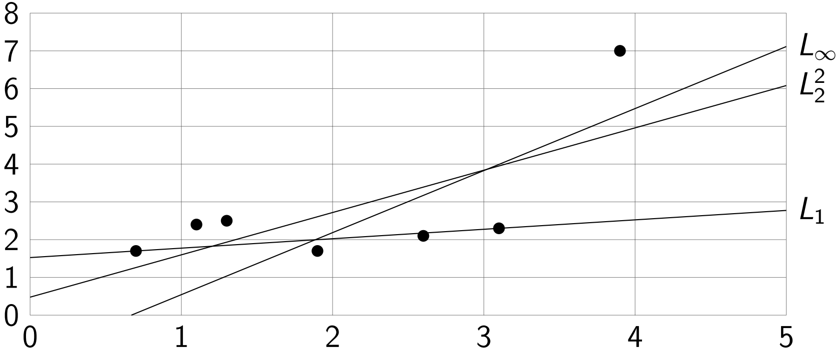

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But that doesn't mean that these predictions minimize the error for future predictions....

Training and Test Sets

To evaluate how well a learner will work on future predictions, we divide the examples into:

- training examples that are used to train the learner
- test examples that are used to evaluate the learner

...these must be kept separate.

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- Why? A probability of zero means "impossible" and has infinite cost if there is one true case in test set.
- Solution: (Laplace smoothing) add (non-negative) pseudo-counts to the data.
  Suppose n_i is the number of examples with $X = v_i$, and c_i is the pseudo-count:

$$P(X = v_i) = \frac{c_i + n_i}{\sum_{i'} c_{i'} + n_{i'}}$$

- Pseudo-counts convey prior knowledge. Consider: "how much more would I believe v_i if I had seen one example with v_i true than if I has seen no examples with v_i true?"