Learning Objectives

At the end of the class you should be able to:

- explain how cycle checking and multiple-path pruning can improve efficiency of search algorithms
- explain the complexity of cycle checking and multiple-path pruning for different search algorithms
- justify why the monotone restriction is useful for A^* search
- predict whether forward, backward, bidirectional or island-driven search is better for a particular problem
- demonstrate how dynamic programming works for a particular problem

Summary of Search Strategies

Strategy	Frontier Selection	Complete	Halts	Space
Depth-first	Last node added			
Breadth-first	First node added			
Best-first	Global min $h(p)$			
Lowest-cost-first	Minimal $cost(p)$			
A^*	Minimal $f(p)$			

Complete — if there a path to a goal, it can find one, even on infinite graphs.

Halts — on finite graph (perhaps with cycles).

Space — as a function of the length of current path

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Depth-first	Last node added	No	No	Linear
Breadth-first	First node added	Yes	No	Exp
Best-first	Global min $h(p)$	No	No	Exp
Lowest-cost-first	Minimal $cost(p)$	Yes	No	Exp
A^*	Minimal $f(p)$	Yes	No	Exp

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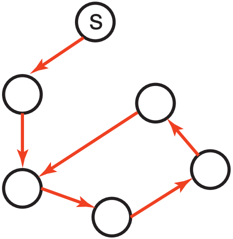
Cycle Pruning

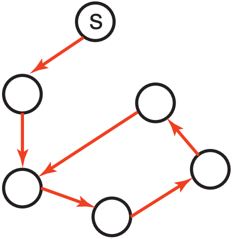


- A searcher can prune a path that ends in a node already on the path, without removing an optimal solution.

Graph searching with cycle pruning

Input: a graph,
a set of start nodes,
Boolean procedure $goal(n)$ that tests if n is a goal node.
$frontier := \{\langle s \rangle : s \text{ is a start node}\}$
while *frontier* is not empty:
select and **remove** path $\langle n_0, \ldots, n_k \rangle$ from *frontier*
if $n_k \notin \{n_0, \ldots, n_{k-1}\}$:
if $goal(n_k)$:
return $\langle n_0, \ldots, n_k \rangle$
$Frontier := Frontier \cup \{\langle n_0, \ldots, n_k, n \rangle : \langle n_k, n \rangle \in A\}$

Cycle Pruning



- In depth-first search, checking for cycles can be done in ________ time in path length.

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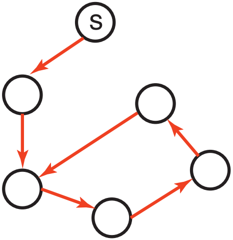
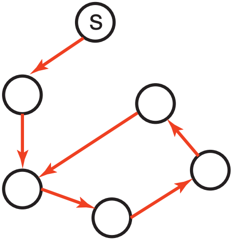


- In depth-first search, checking for cycles can be done in constant time in path length.

Cycle Pruning



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- For other methods, checking for cycles can be done in _____ time in path length.

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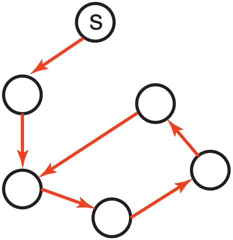

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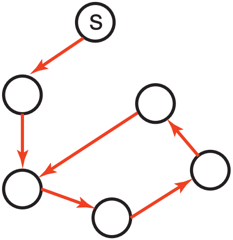
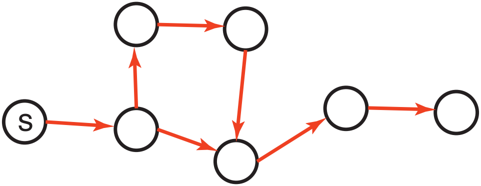


- In depth-first search, checking for cycles can be done in <u>constant</u> time in path length.
- For other methods, checking for cycles can be done in <u>linear</u> time in path length.
- With cycle pruning, which algorithms halt on finite graphs?

Multiple-Path Pruning



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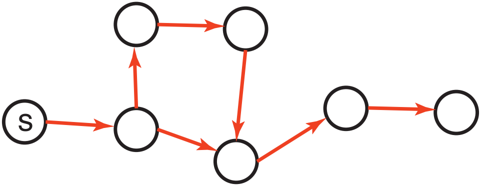

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- What needs to be stored?

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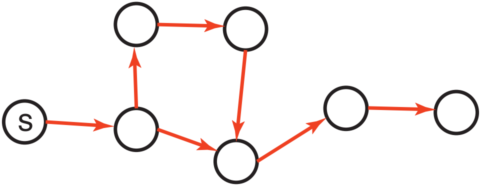

- Multiple path pruning: prune a path to node *n* that the searcher has already found a path to.
- What needs to be stored?
- Lowest-cost-first search with multiple-path pruning is Dijkstra's algorithm, and is the same as A^* with multiple-path pruning and a heuristic function of 0.

Graph searching with multiple-path pruning

Input: a graph,
a set of start nodes,
Boolean procedure $goal(n)$ that tests if n is a goal node.
$frontier := \{\langle s \rangle : s \text{ is a start node}\}$
$expanded := \{\}$
while *frontier* is not empty:
    select and **remove** path $\langle n_0, \ldots, n_k \rangle$ from *frontier*
    if $n_k \notin expanded$:
        add n_k to *expanded*
        if $goal(n_k)$:
            return $\langle n_0, \ldots, n_k \rangle$
        $Frontier := Frontier \cup \{\langle n_0, \ldots, n_k, n \rangle : \langle n_k, n \rangle \in A\}$

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- How does multiple-path pruning compare to cycle pruning?
- Which search algorithms with multiple-path pruning always halt on finite graphs?
- What is the time overhead of multiple-path pruning?
- What is the space overhead of multiple-path pruning?
- Is it better for depth-first or breadth-first searches?
- Can multiple-path pruning prevent an optimal solution being found?

Multiple-Path Pruning & Optimal Solutions

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Multiple-Path Pruning & Optimal Solutions

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- remove all paths from the frontier that use the longer path.
- change the initial segment of the paths on the frontier to use the lower-cost path.
- ensure this doesn't happen. Make sure that the lower-cost path to a node is expanded first.

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- Suppose path p' ends at node n'.

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- Suppose $cost(n', n)$ is the actual cost of a path from n' to n. The path to n via p' has a lower cost that p so:

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$$cost(n', n) < cost(p) - cost(p') \leq h(n') - h(n).$$

We can ensure this doesn't occur if
$|h(n') - h(n)| \leq cost(n', n)$.

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- If h satisfies the monotone restriction, A^* with multiple path pruning always finds a least-cost path to a goal.
- This is a strengthening of the admissibility criterion.

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- Note: when graph is dynamically constructed, the backwards graph may not be available. One might be more difficult to compute than the other.

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  - in the other direction, another method (typically depth-first) can be used to find a path to these interesting states.
  - How much is stored in the breadth-first method, can be tuned depending on the space available.

Island Driven Search

- Idea: find a set of islands between s and g.

$$s \longrightarrow i_1 \longrightarrow i_2 \longrightarrow \ldots \longrightarrow i_{m-1} \longrightarrow g$$

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- The subproblems can be solved using islands $\Longrightarrow$ hierarchy of abstractions.

Dynamic Programming

Idea: for statically stored graphs, build a table of $dist(n)$ the actual distance of the shortest path from node n to a goal.
This can be built backwards from the goal:

$$dist(n) = \begin{cases} 0 & \text{if } is\_goal(n), \\ \min_{\langle n,m \rangle \in A}(|\langle n, m \rangle| + dist(m)) & \text{otherwise.} \end{cases}$$

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- This can be used locally to determine what to do from any state.
- Why not use A^*?
- There are two main problems:
  - It requires enough space to store the graph.
  - The *dist* function needs to be recomputed for each goal.